

Static and Dynamic Performance of Journal Bearings lubricated With Nano-Lubricant

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Abstract:-

In this paper, a suitable mathematical model was proposed to investigate the effect of adding nanoparticles to the base oil lubricating the cylindrical journal bearing on the dynamic characteristics of the bearing system. Suitable time dependent Reynold's equation has been perturbed in order to evaluate the static and dynamic characteristics of the journal bearing. Modified Krieger and Dougherty viscosity model has been used to investigate the oil viscosity variation with the particle concentration of the Nano-particles. A suitable computer program has been written in MATLAB to solve the governing equations. Titanium dioxide Nano-particles have been added to the base oil with different particle concentrations (0.1%,0.5%,1%, 1.5% and 2%) in order to enhance its viscosity. The computer programs has been validated by comparing some of the results calculated in this work with other published researches and it was found that the discrepancy is less than 2%.The investigations reveal that the change in oil viscosity due to the addition of the nanoparticles to the base oil has a pronounced effect on the linear dynamic coefficients. An increase in linear stiffness coefficient up to 20% has been obtained when the bearing lubricated with nanolubricant that has 0.5% particle concentration of the nanoparticles while the linear damping coefficients increase up to 31% for the bearing works at the same conditions.

Keywords: Reynolds equation, Journal Bearing, Nano Lubricant, dynamic coefficient , perturbation technique.

Introduction:-

Journal bearing is a machine element used to support radial loads. The externally applied load on the bearing can be supported by the hydrodyn-amical reaction lift force.

A cylindrical journal bearing is a conventional type of hydrodynamic bearing which can suffer from a specific type of vibration of large lateral amplitude because of the existing of self instability.[6] presented a numerical method for

calculating the stiffness and damping coefficients. A first order of small amplitude perturbation was used to determine the dynamic coefficients, the resulting equations were solved by using FDM and successive over-relaxation method. [7] investigated the relationship between eight linear oil-film force coefficients of circular journal bearings and the perturbation amplitudes. The force coefficients were calculated by the finite perturbation method and compared with those calculated by the infinitesimal perturbation method. Numerical experiments show that the calculated results from both finite perturbation and infinitesimal perturbation methods are very close when the perturbation amplitudes are less than for normal bearing eccentricities. [8] theoretically analyzed the dynamic characteristics of a journal bearing with two axial grooves one at the top and the other at the bottom. The perturbation technique was used to evaluate the dynamic coefficients and stability. It has been shown that the stability of bearing improves for smaller groove angle and length.[3] proposed a method includes transforming of the perturbed Reynolds equations into finite element equations to calculate the dynamic coefficients (stiffness and the damping coefficients) of the coupled journal and thrust bearings. Verification of the obtained results has been carried out by comparing the dynamic coefficient obtained by the proposed method with that

obtained by numerical differentiation of the bearing force with respect to finite displacements and finite velocities of bearing center. The verification shows that the proposed method can be used to calculate the dynamic coefficients effectively. [8] investigated the static and dynamic characteristics of axial grooved journal bearing at different locations theoretically. A first-order perturbation method for each location of the groove has been used to calculate the dynamic coefficients and the stability. The stiffness and damping coefficient magnitude is found to be higher for the bearing with smaller groove angle and groove length.[4] presented static and dynamic analysis to the performance characteristics of a journal bearing lubricated with nano lubricant. The dynamic characteristics are studied through threshold speed and damped frequency. Copper oxide CuO cerium oxide CeO_2 and aluminium oxide Al_2O_3 nanoparticles were used. The static and dynamic performance characteristics of journal bearing are computed for various values of eccentricity ratios for isoviscous and thermoviscous lubricants. The results show that the addition of nanoparticles to the base oil has no significant effect on the performance of the bearing in the isoviscous case while it became significant in thermoviscous case. [5] studied the effect of ultra-fine additives on hydrodynamic

lubrication in journal bearings theoretically and experimentally. Lubricants samples containing the fullerene black, fullerenes, molybdenum disulfide and fluoropolymerin with particle concentration not higher than 0.05% of the mass in base low viscosity mineral oil were tested. The experiment was made during the run-down of the rotor, and the friction coefficient in the journal bearing and the vibrations of its housing were measured. The results obtained show that ultra-fine additives significantly decreased the load carrying capacity and the friction coefficient in the journal bearing, as well as the vibration of the bearing housing. It can be clearly shown from the above presentation that there is a rare work related to the effect of using nanolubricant on the dynamic characteristics of journal bearings which is the main goal of the present work.

Theoretical Analysis

The approach proposed by [10] is adopted to evaluate the dynamic coefficients which give a complete description to the dynamic characteristics of the journal bearing. This method depends on the perturbation of the journal center about its equilibrium position and using the partial derivative method to calculate the linear dynamic coefficients. Figure (1), shows the geometry and the coordinates system

of the journal bearing. The following time dependent Reynolds equation is utilized in this paper, [2].

$$\begin{aligned} & \frac{\partial}{\partial x} \left(\rho h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\rho h^3 \frac{\partial p}{\partial y} \right) \\ & = 6\mu_0 U \frac{\partial(\rho h)}{\partial x} + 12\mu_0 \frac{\partial(\rho h)}{\partial(t)} \end{aligned} \quad (1)$$

Where

ρ =oil density (kg/m³)

μ_0 = base oil viscosity(Pa.s)

The oil film thickness of an aligned journal bearing can be expressed as

$$h = c + e \cos \theta \quad (2)$$

In order to generalize equation (1), it can be normalized by using the following non-dimensional variables:

$$\begin{aligned} \theta &= \frac{x}{R} & \bar{z} &= \frac{z}{L/2} & \tau &= \omega t \\ \bar{h} &= \frac{h}{c} & \bar{P} &= \frac{c^2}{6\mu_0 \omega R^2} p \end{aligned}$$

Hence equation (1) can be rewritten as:

$$\begin{aligned} & \frac{\partial}{\partial \theta} \left(\bar{h}^3 \frac{\partial \bar{P}}{\partial \theta} \right) + \left(\frac{D}{L} \right)^2 \frac{\partial}{\partial \bar{z}} \left(\bar{h}^3 \frac{\partial \bar{P}}{\partial \bar{z}} \right) \\ & = \bar{\mu} \left(\frac{\partial \bar{h}}{\partial \theta} + 2 \frac{\partial \bar{h}}{\partial \tau} \right) \end{aligned} \quad (3)$$

where

$\bar{\mu} = \frac{\mu_n}{\mu_0}$ = non-dimensional viscosity

μ_n = Nano-lubricant viscosity (Pa.s)

μ_0 = base oil viscosity (Pa.s)

D= journal diameter (m)

L =journal length(m)

The oil film thickness in nondimensional form can be written as,

$$\bar{h} = 1 + \epsilon \cos \theta \quad (4)$$

Where

$\epsilon = \frac{e}{c}$ = eccentricity ratio

θ = angular position(deg.)

The Nano-lubricant viscosity can be evaluated in this case by using the following modified Kriger and Dougherty viscosity model [1].

$$\bar{\mu} = \left(1 - \frac{\varphi}{0.605} \left(\frac{a_0}{a} \right)^{1.2} \right)^{-1.51} \quad (5)$$

where

$\frac{a_0}{a}$ = the aggregate ratio which is equal to 7.77 for TiO_2 .

φ = Particle concentration

Perturbation Technique:

The time dependent Reynolds equation (1) has been perturbed by assuming the eccentricity ratio and attitude angle at the equilibrium position, i.e. ϵ_0 , and ψ_0 are satisfied. The eccentricity ratio can be perturbed as [10].

$$\epsilon = \epsilon_0 + E_0 e^{i\Omega\tau} \quad (6)$$

While the attitude angle can be perturbed as:

$$\psi = \psi_0 + \Psi_0 e^{i\Omega\tau} \quad (7)$$

Where(o) refers to the equilibrium position. Substituting equation (6)

and (7) in (4) to obtain the perturbed oil film thickness as:

$$\bar{h} = \bar{h}_0 + \bar{h}_1 e^{i\Omega\tau} \quad (8)$$

$$= \bar{h}_0 + (E_0 c \cos \theta + \epsilon_0 \psi_0 \sin \theta) e^{i\Omega\tau}$$

The perturbed hydrodynamic pressure can be expressed as, [10].

$$\bar{P} = \bar{P}_0 + \bar{Q}_{10} e^{i\Omega\tau} + \bar{Q}_{20} e^{2i\Omega\tau} + \dots \quad (9)$$

where

\bar{P} = Perturbed hydrodynamic pressure

\bar{P}_0 = steady state pressure

Q_i ($i = 10, 20, \dots$) = complex variables which represent the amplitudes of the first, second, etc., terms of dynamic oil film pressure. (In the present work, the first two dynamic pressure components are taken into consideration).

Substituting equations (8) and (9) into equation (3) and collecting the zero, one and two pressure terms respectively while neglecting the higher order terms to get the following set of second order non-linear partial differential equations [10].

$$\frac{\partial}{\partial \theta} \left(\bar{h}^3 \frac{\partial \bar{P}_0}{\partial \theta} \right) + \left(\frac{D}{L} \right)^2 \frac{\partial}{\partial \bar{z}} \left(\bar{h}^3 \frac{\partial \bar{P}_0}{\partial \bar{z}} \right) = \bar{\mu} \frac{\partial \bar{h}}{\partial \theta} \quad (10)$$

$$\begin{aligned} & \frac{\partial}{\partial \theta} \left(\bar{h}_0^3 \frac{\partial \bar{P}_1}{\partial \theta} \right) \\ & + \left(\frac{D}{L} \right)^2 \frac{\partial}{\partial \bar{z}} \left(\bar{h}_0^3 \frac{\partial \bar{P}_1}{\partial \bar{z}} \right) \\ & + \frac{\partial}{\partial \theta} \left(3\bar{h}_0^2 \cos \theta \frac{\partial \bar{P}_0}{\partial \theta} \right) \\ & + \left(\frac{D}{L} \right)^2 \frac{\partial}{\partial \bar{z}} \left(3\bar{h}_0^2 \cos \theta \frac{\partial \bar{P}_0}{\partial \bar{z}} \right) \\ & = \bar{\mu} (-\sin \theta + 2i\Omega \cos \theta) \end{aligned} \quad (11)$$

$$\begin{aligned} & \frac{\partial}{\partial \theta} \left(\bar{h}_0^3 \frac{\partial \bar{P}_2}{\partial \theta} \right) \\ & + \left(\frac{D}{L} \right)^2 \frac{\partial}{\partial \bar{z}} \left(\bar{h}_0^3 \frac{\partial \bar{P}_2}{\partial \bar{z}} \right) \\ & + \frac{\partial}{\partial \theta} \left(3\bar{h}_0^2 \sin \theta \frac{\partial \bar{P}_0}{\partial \theta} \right) \\ & + \left(\frac{D}{L} \right)^2 \frac{\partial}{\partial \bar{z}} \left(3\bar{h}_0^2 \sin \theta \frac{\partial \bar{P}_0}{\partial \bar{z}} \right) \\ & = \bar{\mu} (\cos \theta + 2i\Omega \sin \theta) \end{aligned} \quad (12)$$

where \bar{P}_0, \bar{P}_1 and \bar{P}_2 are the static, type 1 and 2 dynamic pressures respectively. The static characteristics of the journal bearing lubricated with Nano-lubricant can be evaluated by solving equation (10), using the following Reynolds boundary conditions

$$\bar{P}_0 = \frac{\partial \bar{P}_0}{\partial \theta} = 0 \quad \text{at} \quad \theta = \theta_c$$

θ_c = angle at which cavitation starts

A finite difference method with successive over relaxation scheme has been used to solve such an equation. The components of the hydrodynamic forces can be calculated by integrating the static hydrodynamic pressure over the bearing surface as follows:

$$\begin{aligned} \bar{F}_{x0} &= \int_0^1 \int_0^{2\pi} \bar{P}_0 \cos \theta \, d\theta \, d\bar{z} \\ \bar{F}_{y0} &= \int_0^1 \int_0^{2\pi} \bar{P}_0 \sin \theta \, d\theta \, d\bar{z} \end{aligned} \quad (13)$$

The static load carried by the bearing can be calculated as:

$$\bar{F} = \sqrt{\bar{F}_{x0}^2 + \bar{F}_{y0}^2} \quad (14)$$

While the Sommerfeld number can be calculated as

$$SN = \frac{1}{\pi \bar{F}} \quad (15)$$

The attitude angle is calculated as:

$$\psi = -\tan^{-1} \frac{\bar{F}_y}{\bar{F}_x} \quad (16)$$

Dynamic Coefficients

The determination of the dynamic characteristics of the journal bearing requires the solution of equations (11) and (12). For this purpose the equilibrium position of the journal center must be calculated first. The essential condition to specify the equilibrium position of the journal center is when the horizontal component of the hydrodynamic force, \bar{F}_{y0} equal to zero while the vertical component \bar{F}_{x0} equal the applied (external) load. The same technique adopted to solve equation (10) was used to solve equations (11) and (12). The dynamic bearing forces components can be calculated by integrating the dynamic pressures \bar{P}_1 and \bar{P}_2 , over the bearing area as follows:

$$\bar{F}_{x1} = \int_0^1 \int_0^{2\pi} \bar{P}_1 \cos \theta \, d\theta \, d\bar{z} \quad (17)$$

$$\bar{F}_{y1} = \int_0^1 \int_0^{2\pi} \bar{P}_1 \sin \theta \, d\theta \, d\bar{z}$$

$$\bar{F}_{x2} = \int_0^1 \int_0^{2\pi} \bar{P}_2 \cos \theta \, d\theta \, d\bar{z} \quad (18)$$

$$\bar{F}_{y2} = \int_0^1 \int_0^{2\pi} \bar{P}_2 \sin \theta \, d\theta \, d\bar{z}$$

The dynamic stiffness and damping coefficients can be defined as the derivatives of the forces with respect to the displacements and velocities as follows:

$$K_{xx} = \frac{\partial F_x}{\partial x} \quad K_{xy} = \frac{\partial F_x}{\partial y} \quad K_{yx} = \frac{\partial F_y}{\partial x} \\ K_{yy} = \frac{\partial F_y}{\partial y} \quad (19)$$

$$D_{xx} = \frac{\partial F_x}{\partial \dot{x}} \quad D_{xy} = \frac{\partial F_x}{\partial \dot{y}} \quad D_{yx} = \frac{\partial F_y}{\partial \dot{x}} \\ D_{yy} = \frac{\partial F_y}{\partial \dot{y}} \quad (20)$$

Hence, the linear dynamic coefficients can be expressed as:

$$\bar{K}_{xx} = -Re(\bar{F}_{x1}) \quad \bar{K}_{yx} = -Re(\bar{F}_{y1}) \\ \bar{K}_{xy} = -Re(\bar{F}_{x2}) \quad \bar{K}_{yy} = -Re(\bar{F}_{y2}) \\ \bar{D}_{xx} = -Im(\bar{F}_{x1}) \quad \bar{D}_{yx} = -Im(\bar{F}_{y1}) \\ \bar{D}_{xy} = -Im(\bar{F}_{x2}) \quad \bar{D}_{yy} = -Im(\bar{F}_{y2}) \quad (21)$$

Results and Discussion

The results obtained in the present work are for the bearing with following geometric and operating conditions

Journal length(L)=40mm

Journal diameter(D)=40mm

Radial clearance(c)=0.15mm

Oil viscosity= 0.018Pa.s

Journal rotational speed=6000rpm

Groove angle (β)=20°

To verify the mathematical model used in this study, a comparison of the calculated results in the present work for the attitude angle and Sommerfeld number at the equilibrium position with that obtained by [6] is presented in figure (2). Both the Sommerfeld and the attitude angle have been drawn against the eccentricity ratio. This figure clearly depicts the good agreement between the results.

Another validation has been carried out and presented in figure (3-a and b). This figure shows the direct and cross coupled linear stiffness and damping coefficients against the Sommerfeld number. These figures show a comparison for these coefficients calculated in the present work with that obtained by [6]. The discrepancy between the results has been calculated and found to be less than 2%. The validation leads to the main conclusion that the computer programs prepared to study the problem of the present work can be used with acceptable confidence. Some of the most important bearing static characteristics have been presented in figure (4). Figure (4-a) shows the results of Sommerfeld number against the full range of eccentricity ratio of the bearing. It is clear that the Sommerfeld number decreases when the bearing works at higher eccentricity ratios. This can be attributed to the high hydrodynamic pressure generated in this case. This figure also shows that the effect of adding nanoparticles to the base oil is to decrease the Sommerfeld number which refer to the increase in load carrying capacity of the bearing in this case. The percentage decrease in Sommerfeld number was calculated and found to be 8% when the bearing lubricated with nano-lubricant that contains 0.5% TiO_2 nanoparticles while it becomes 27% when the bearing lubricated with nano-lubricant that has 1% particle

concentration of the same nanoparticles. These percentages were calculated for a bearing working at an eccentricity ratio of 0.5. This can be explained by knowing that the oil viscosity of the base oil increases with the addition of nanoparticles. The higher the percentage of the nano-particles added to the base oil, the higher the decrease in the Sommerfeld number. This figure also shows that a little effect of the nanoparticles was noticed when it is added with small percentage since the oil seems to behave like pure oil. Figure (4-b) shows that the load carrying capacity of the bearing increases when the bearing lubricated with nano-lubricant. The higher the percentage of the nanoparticles in the oil, the higher is the bearing load carrying capacity. The increasing percentage in load carrying capacity was calculated for a bearing working at eccentricity ratio of 0.5 lubricated with nano-lubricant contains 0.5% and 1% particle concentrations. It was found to be 34% and 50% respectively. The bearing load carrying capacity becomes higher when the bearing works at higher eccentricity ratios. The effect of lubricating the bearing with oil containing nanoparticles of different particle concentrations on stiffness coefficients can be shown in figures (5-a,b and c). The direct stiffness coefficient (K_{xx}) and (K_{yy}) obtained in the present study has been presented against the full range of

the bearing eccentricity ratios. It is clear that the stiffness coefficients (K_{xx} and K_{yy}) increase when the bearing works at higher eccentricity ratios. This can be depicted by the high hydrodynamic load carried by the bearing in this case. Also this figure shows that higher increase in (K_{xx} and K_{yy}) can be obtained when the bearing lubricated with oil containing nanoparticles. The higher the particle concentration in the base oil, the higher is the stiffness coefficient mentioned above. This can be explained by the change of the physical properties of the nano-lubricant especially its viscosity which became higher than the viscosity of the base oil, hence, causes higher load carrying capacity and higher oil film stiffness. The percentage increase in (K_{xx} and K_{yy}) has been calculated and found to be 15.3% and 3.4% for a bearing working at eccentricity ratio of 0.5 lubricated with nano-lubricant that has nanoparticle concentrations of 0.5% in comparison with that lubricated with pure oil. The cross coupled stiffness (K_{yx}) shows a negative values when the bearing works at an eccentricity ratios less than 0.7 while it became positive when the bearing works at an eccentricities higher than 0.7 as can be shown from figure (5-b). This figure depicts that the negative values of the (K_{yx}) slightly increases for the bearing works at eccentricity ratios less than 0.6 after that it decreases until the bearing

eccentricity ratio 0.7 when its values became increasing positive. Figures (6-a) to (6-c) show the behaviour of the direct and cross coupled damping coefficients against the full range of the eccentricity ratios. It can be seen from this figure that the damping ratios (D_{xx} , $D_{xy} = D_{yx}$ & D_{yy}) always increase when the bearing works at higher eccentricity ratios. This can be attributed to the high load carried by the bearing in this case.

Conclusions:-

1. The load carrying capacity of the bearing increases while the Somerfeld decreases by the addition of TiO_2 nanoparticles to the base oil. An increase of 34% in load carrying capacity while a decrease of 8% in Somerfeld number has been obtained for the bearing lubricated with nanolubricant that contains 0.5% of such nano-particles.
2. Bearing Dynamic stiffness coefficients K_{xx}, K_{yy} are increased by 15.3% and 3.4% while K_{yx} increases by 20% when the bearing lubricated with oil containing 0.5% TiO_2 nano-particles and works at an eccentricity ratio of 0.5.
3. Bearing Dynamic damping coefficients D_{xx} and D_{yy} are increased by 31.5% and 17.6 % while D_{yx} increases by 16.2% when the bearing lubricated with oil containing 0.5% TiO_2 nano-particles and works at an eccentricity ratio of 0.5.

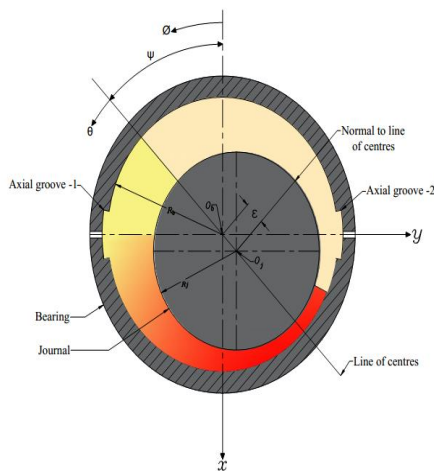


Fig. 1 Journal bearing geometry

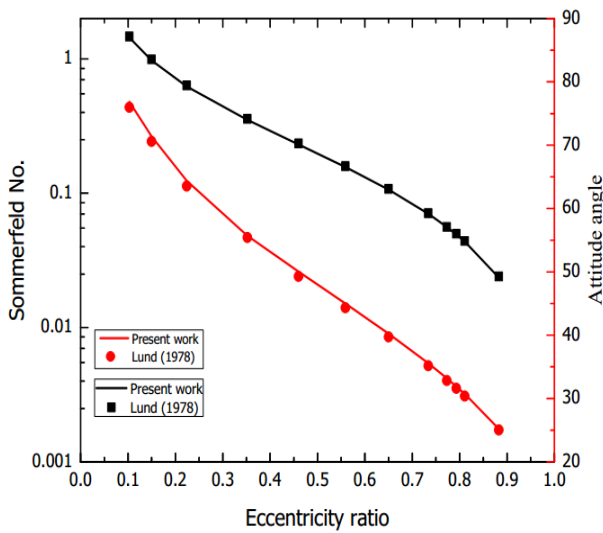
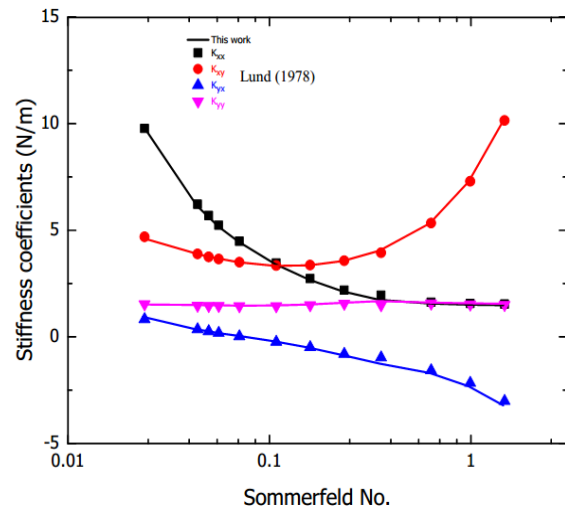
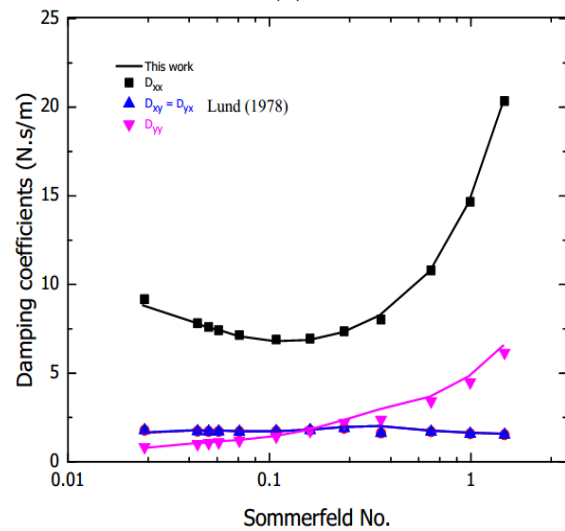


Fig. 2 Static validation

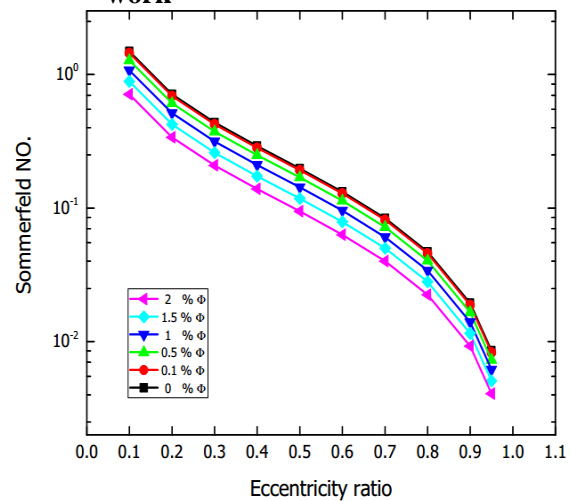


(a)



(b)

Fig.3 Validation of the dynamic coefficients obtained in the present work



(a)

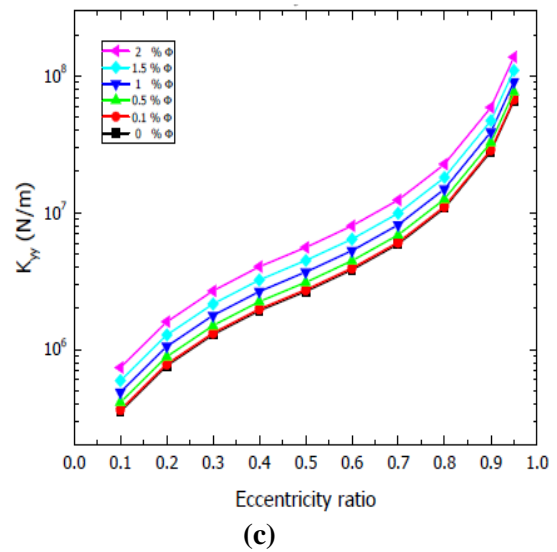
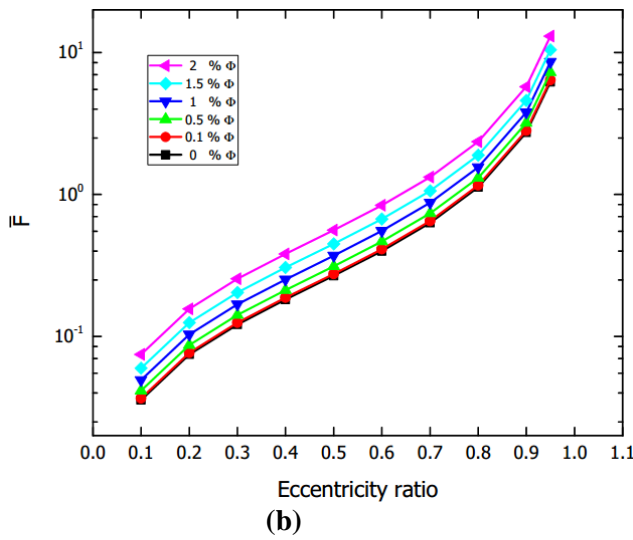
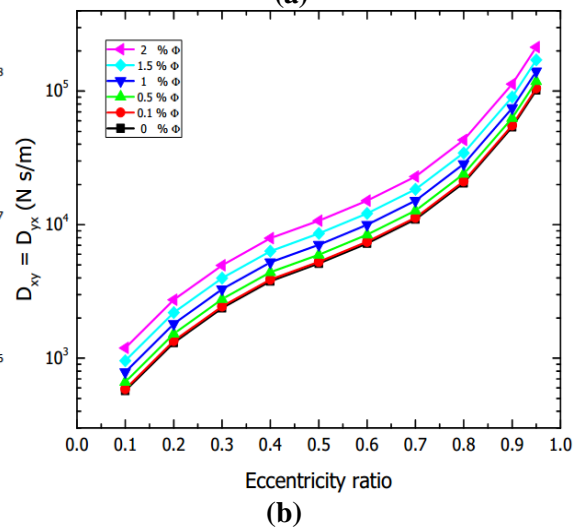
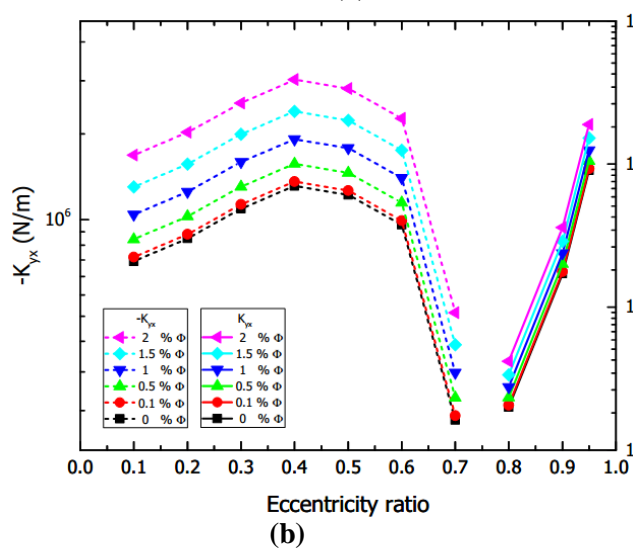
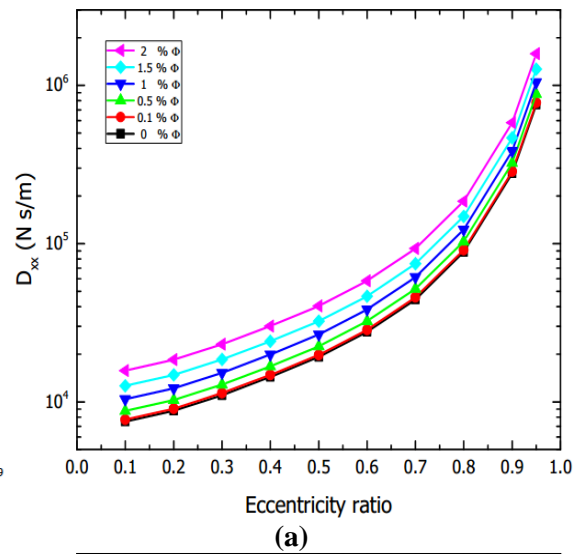
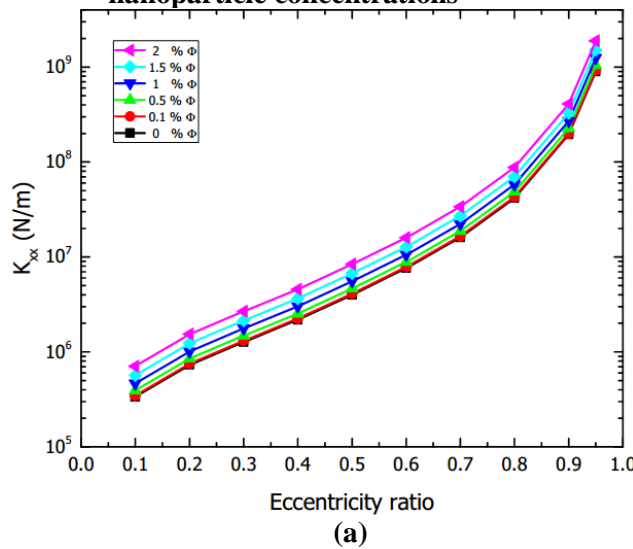


Fig. 4: Static parameters for different nanoparticle concentrations

Fig. 5 Linear stiffness coefficients as a function of eccentricity ratios and different nanoparticle concentrations.



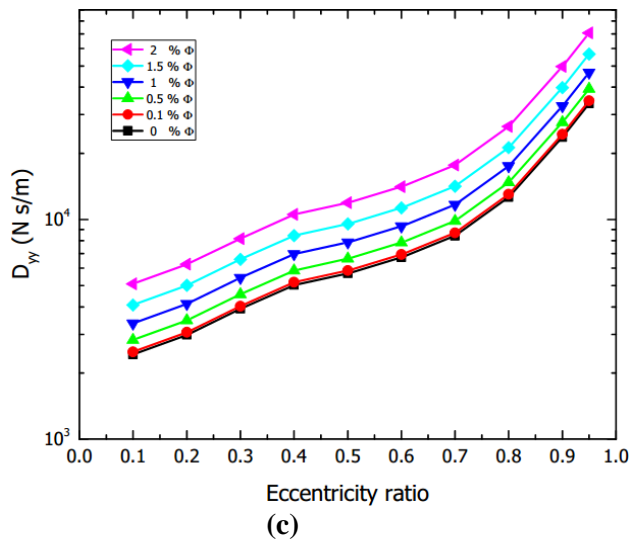


Fig.6 Linear damping coefficients for different nanoparticle concentrations

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الخصائص السكونية والحركية للمساند المقعدية المزيتة بزيوت حاوية على دقائق متناهية في الصغر

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الخلاصة:

تم في هذا البحث اعداد نموذج رياضي مناسب لدراسة تأثير اضافة دقائق متناهية في الصغر الى الزيت الاساس المستخدم في تزييت المساند المقعدية الاسطوانية على الخصائص الحركية لتلك المساند. استخدمت معادلة رينولدز المعتمدة على الزمن وتجزئتها بشكل مناسب لغرض تقييم الخصائص السكونية والحركية للمسند بينما استخدمت معادلة (Krieger و Dougherty) للاخذ بنظر الاعتبار تأثير اضافة الدقائق المتناهية في الصغروبنسب مختلفة على لزوجة الزيت الاساس. حلت المعادلات الحاكمة عن طريق بناء برنامج مناسب باستخدام ال(MATLAB) . تم دراسة تأثير اضيفت دقائق متناهية في الصغر من مادة ثاني اوكسيد التيتانيوم (TiO_2) الى الزيت الاساس وبنسب وزنية بلغت (0.1%, 0.5%, 1.5%, 2%) . حققت صحة البرنامج الذي تم اعداده لحل المعادلات الحاكمة عن طريق مقارنة بعض النتائج المستحصلة في العمل الحالي مع تلك المستحصلة من قبل الباحثين (Lund) و (Thomson) سنة 1978. احتسبت نسبة الخطأ بين النتائج ووجدت بانها اقل من 2%. اظهرت النتائج المستحصلة في هذا البحث ان اضافة الدقائق المتناهية في الصغر المذكورة لها تأثير واضح على المعاملات الحركية الخطية حيث اظهرت معاملات المرونة الخطية زيادة تصل الى 20% عند تزييت المسند المقعدى بزيوت حاوي على تلك الدقائق وبتركيز يصل الى 0.5% فيما ازدادت معاملات التخميد الخطية بنسبة 31% بالنسبة للمسند الذي يعمل تحت نفس الظروف اعلاه.

الكلمات المفتاحية: معادلة رينولدز، المسند المقعدى، الزيوت الحاوية على دقائق متناهية في الصغر، المعاملات الديناميكية، تقنية الاضطراب