Effect of Leaf Spring Curvature and Shape on Its Static and Dynamic Performance

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Abstract: -

Transportation luxurious conditions haven't been known before the appearance of spring and damper as cooperative parts within the suspension systems. Spring shapes take a lot of forms depending on their duties and the working conditions as well as the available space. Due to its capability to absorb large amount of shock, beam bending mechanism has been adopted in cargo vehicle suspension systems. This work proposes circular base plate shape simply supported laminated leaf springs distributed according to set of suggested curvatures (circular, second order parabola and exponential) as geometrical modifications to increase the load carrying capacity by minimizing the induced maximum bending stress, enhancing the fatigue life or the dynamic safety factor and investigate their effects on the vibration characteristics. The proposed curvatures have been adopted to be alternative leaf springs instead of classical trapezoidal shape – circular curvature and represented mathematically in polar coordinate and feed to be numerically simulated and solved using Ansys Workbench software ver. 15. The most important results of this work is that the proposed circular shape leaf spring base plate reduces the maximum static bending stress by about 36% regardless its type of curvature and the dynamic safety factor under completely reversed load has been improved by about 300% by using the exponential curvature instead of the classical circular curvature.

Keywords: - Laminated Leaf Springs, Leaf Spring Shape and curvature, Leaf Spring stresses.

1-Introduction

Leaf spring is a curved beam works under bending mechanism such as mono leaf spring or laminated one which is used to absorb the sadden shocks as a parallel springs and pass it gradually to the support frame giving the damper unit the time to dissipate a certain amount of the shock energy [9]. It stands as an efficient opponent among the different spring types because of its ability to absorb energy as a guided ends structure member and it is preferred to be made with maximum strength and minimum modulus of elasticity along its length [8]

In curved beam, the distribution of its curvature and the geometrical characteristics of the cross sectional area identifies its load carrying capacity [7]. The well-known leaf spring shape is the trapezoidal one which ensures a constant bending stress distribution along its length while the circular curvature, according to the previous works, ensures the minimum deflection [5]. Extensive works have dealt with the problem of leaf spring bending stress as a straight beam *regardless of its curvature*. This work adopts different laminated leaf spring curvatures (circular, second order parabola and exponential) and as a result bending stress will be evaluated for both standard leaf spring (trapezoidal shape base plate with circular curvature) The geometrical representation for all of the studied cases has been based on one condition that is the length and initial deflection must be the same and the FEA has taken into account the material, applied load and boundary conditions similarity. The whole study was conducted numerically using Ansys software ver. 15. and analytical verified depending on previous works for special cases.

2- Leaf Spring Geometry and Mechanism:

The classical form of leaf spring is a multiple curved rectangular plates all of the same width and thickness with gradually increased lengths placed in contact to each other where the full length upper leaf is called the master leaf and the other leaves are called the graduated leaves [3].

Under transverse loads, leaf spring flatten to be of greater radius of curvature leading the spring to be stiffer and initiate the relative sliding movement between the successive leaves which is the cause of noise and friction accompanied with the unpaved ways [4]. Previous works proposed the leaf spring base plate shape to be of trapezoidal one of constant thickness as shown in **Fig.1-a** to be cut into a certain number of strips differ in length. These strips are mounted on each other using central bolt and some clamps as shown in **Fig. 1-b**. The trapezoidal shape ensures a constant bending stress along the spring length taking into account the following assumptions [5],[12]

1. Beam deforms in the plane of its curvature.
2. Plane sections remain plane after bending.
3. Beam material is homogeneous and follows Hook's low.
4. There are no residual stresses.
5. No inter leaf friction is considered.

Where the bending stress for simply supported straight leaf spring of trapezoidal shape and its associated deflection is [5]

$$\sigma_b = \frac{3WL}{2nbt^2} \tag{1}$$

$$\delta_{max.} = \frac{3WL^3}{8Enbt^3} \qquad (2)$$

Where:
n: number of leaves, t: thickness of plate, W:applied load, b: width of the plate, L: base plate length, E: modulus of elasticity.

Equation (1) is utilized to evaluate the maximum bending stress for straight trapezoidal leaf spring no matter what type of curvature it has, while equation (2) describes the maximum deflection of circular curvature leaf spring regardless its shape. This work tunes the leaf spring moment of inertia to be more dependent on x direction than the trapezoidal shape of the same dimensions. By adopting a circular shape as a base of the proposed leaf spring shape rather than the trapezoidal one see **Fig. 2-a**. The critical vibration amplitude of the leaf spring has been found to be inversely proportional to its natural frequency [2] where

$$\delta_{Dc} = \frac{g}{{\omega_n}^2} \qquad (3)$$

3- Leaf Spring Geometrical representation:

The general idea of leaf spring design is to use a plate of length-dependent cross sectional area to increase its second moment of area along its length producing lower bending stress levels. The well-known shapes are trapezoidal plate to be cut into a certain number of leaves with knife edge ends leading to stress concentration zones. The representation consists of two parts; the first one replaces the knife edge plates by rectangular plates of the same width and shorter in length (equivalent length L_e) for both shapes i.e. trapezoidal and circular plates see **Fig. 1-b** and **Fig. 2-b**, while the second part aims to introduce different curvatures for each leaf shape see **Fig. 1-c** and **Fig. 2-c**. The main principle to evaluate the equivalent leaf lengths is areas similarity for edge end leafs and rectangular leafs so for trapezoidal plate shape or ***trapezoidal leaf shape***, as in **Fig. 3-a**

$$y = B\left(0.5 - \frac{1}{L} * x\right) \qquad (4)$$

$$A_i = 4\int_{y_1}^{y_2} x * dy \qquad (5)$$

$$A_{ie} = b * L_{ie} \qquad (6)$$

$$A_i = A_{ie} \qquad (7)$$

$$L_{ie} = \frac{4L}{b}\int_{b/2(i-1)}^{(b/2)i}\left(0.5 - \frac{y}{B}\right)dy \qquad (8)$$

Where ***B*** is the base plate width and ***b*** is the leaf spring width, ***A_i*** is the single knife edge leaf area, ***A_e*** and ***L_e*** are the knife edge equivalent rectangular area and length respectively.

For ***Circular leaf shape*** it has been found to be longer than its counterpart trapezoidal one and mathematical representation according to **Fig. 3-b** is

$$x^2 + y^2 = R^2 \tag{9}$$

$$A = 4\int_{y_1}^{y_2} x * dy \tag{10}$$

$$A = 4\int_{y_1}^{y_2} \left(\sqrt{R^2 - y^2}\right) dy \tag{11}$$

$$L_{ie} = \frac{4}{b}\int_{b/2(i-1)}^{(b/2)i} \left(\sqrt{R^2 - y^2}\right) dy \tag{12}$$

Equation (8) and equation (12) represent the corresponding laminated square leaf spring dimensions to the trapezoidal and circular knife edges springs respectively. Most of theories have dealt with the leaf spring as a simply supported straight beam in stress calculation and here it will be investigated, regarding its strength, under different curvature distributions. **Table. 1** refers to the geometrical and mechanical properties of the studied cases.

a. Circular Curvature

For circular leaf profile it is important to be represented in term of polar coordinate (ρ, θ), see **Fig. 4**, as follows

$$\rho = \sqrt{x^2 + y^2} \tag{13}$$

ρ: Radius of curvature

$$x = \frac{L_e}{2} \tag{14}$$

$$y = \rho - \delta \tag{15}$$

By substituting equations (14), (15) in equation (13) the radius of curvature of master leaf will be

$$\rho_1 = \frac{1}{400} * \left(\delta^2 + \frac{L_{1e)}^2}{4}\right) \tag{16}$$

Where δ is the leaf spring initial curvature.

b. Parabola Curvature

The general form of this profile could be written as:

$$y = ax^2 \tag{17}$$

In this type the radius of curvature is variable and could be calculated in terms of 1st and 2nd derivative of the curvature [6]:

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{1.5}}{\frac{d^2y}{dx^2}} \tag{18}$$

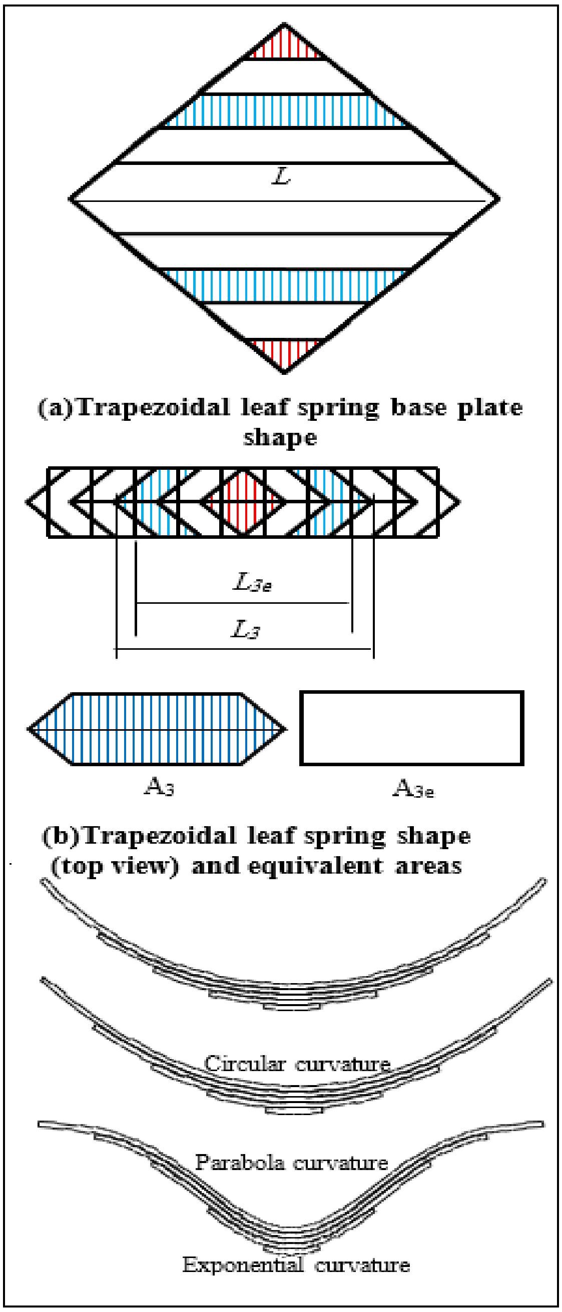
To satisfy the leaf spring geometrical boundary conditions at (x= L/2 ,y = δ = 200) ***a*** is chosen to be 0.8.

By substituting the 1st and 2nd derivative of equation (17)in equation (18) the master leaf radius of curvature the parabola leaf spring will be

$$\rho_1 = \frac{[1+4a^2x^2]^{1.5}}{2a} \tag{19}$$

c. Exponential Curvature

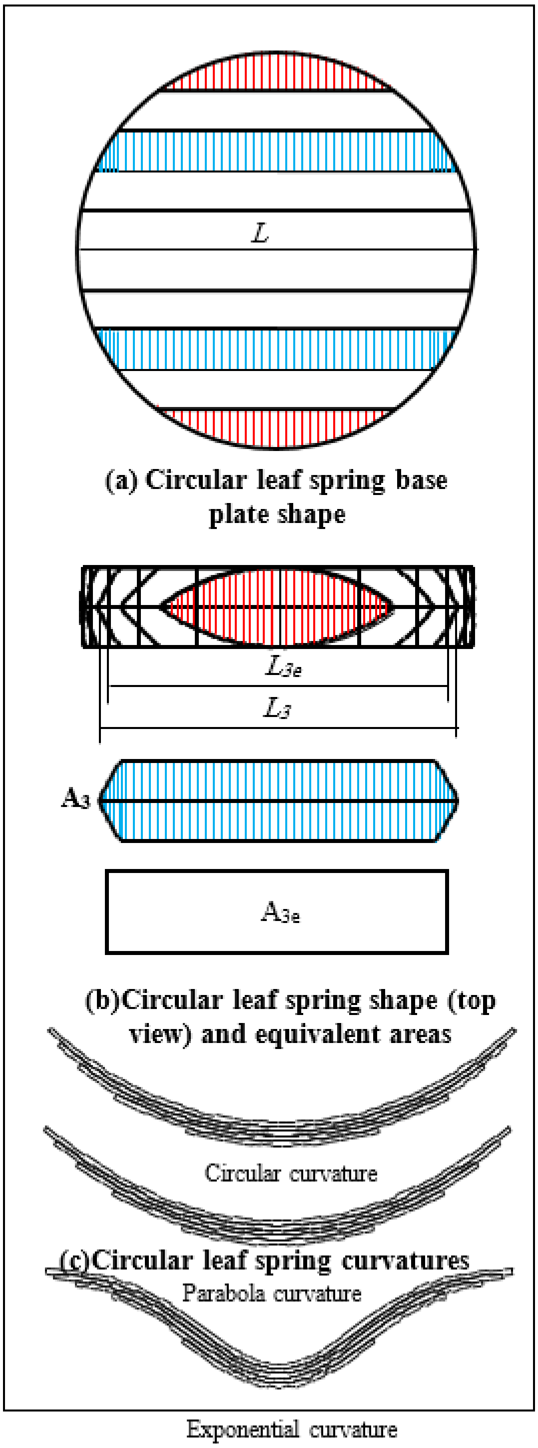
The proposed geometry of this curvature is illustrated in **Fig. 4** while its mathematical representation is, [6]



(a)Trapezoidal leaf spring base plate shape

(b)Trapezoidal leaf spring shape (top view) and equivalent areas

Fig.1 Trapezoidal shape leaf spring different distributions



(a) Circular leaf spring base plate shape

(b)Circular leaf spring shape (top view) and equivalent areas

(c)Circular leaf spring curvatures

Fig. 2 Proposed circular shape leaf spring different distributions

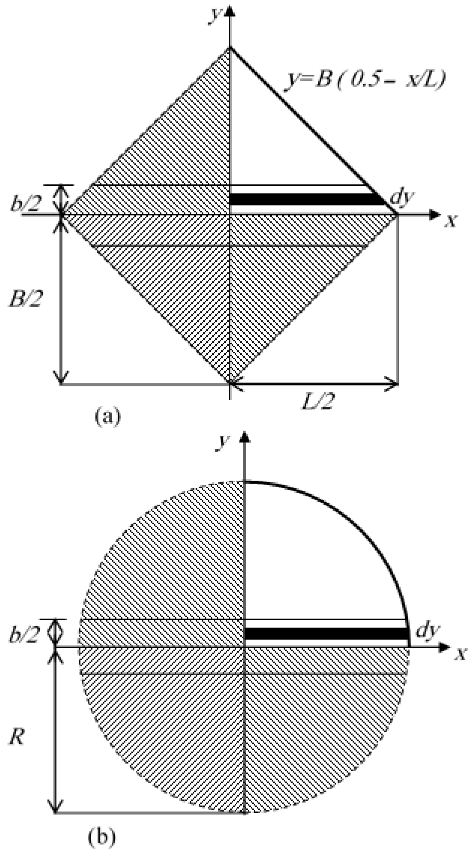


Fig.3 individual leaf spring evaluation, a) Trapezoidal leaf, b) circular leaf

$$y = a * (1 - e^{-(\frac{x^2}{2k^2})}) \qquad (20)$$

Where k is chosen to be 6th of the equivalent length in order to achieve the initial deflection of 200 mm yielding

$$= \delta - \delta * e^{\left(-0.5\left(\frac{x}{L_{ie}/6}\right)^2\right)} \qquad (21)$$

Table 1. Dimensions and mechanical properties of the studied steel leaf spring [10].

Dimension	*Value*
L	1000 mm
B	1000 mm
b	200 mm
t	10 mm
n	5
W	2000 N
δ	200 mm
E	200 N/mm^2
ν	0.3
σ_e	86 N/mm^2
σ_y	250 N/mm^2

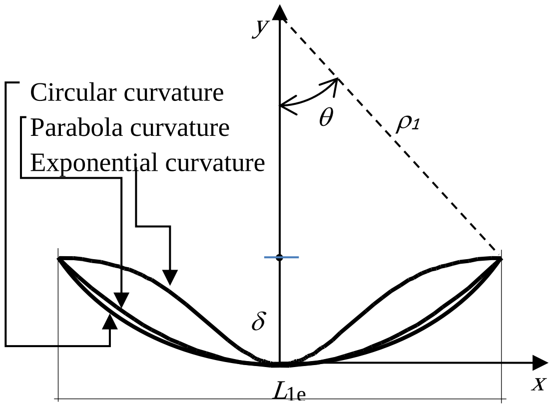


Fig. 4 different leaf spring curvature representations

$$y = \delta - \delta * e^{\left(-2\left(\frac{3x}{L_{1e}}\right)^2\right)} \qquad (22)$$

$$\rho_1 = \frac{\left[1+\delta^2\left(\frac{36x}{L_{1e}}\right)^2 \; e^{-\left(\frac{6x}{L_{1e}}\right)^2}\right]^{1.5}}{\frac{36\delta}{L_{1e}{}^2}\delta * e^{\left(-2\left(\frac{3x}{L_{1e}}\right)^2\right)}\left[1-\frac{36x^2}{L_{1e}{}^2}\right]} \qquad (23)$$

Equations (16, 19 and 23) traced the radius of curvatures at any point of the curvature for the circular , parabola and exponential curvatures respectively. These equations represent the starting point of the geometrical representation which describe the master leaf spring distribution and will be calculated for the graduated leaves by adding the base plate thickness to the master leaf radius of curvature to be

$$\rho_{i+1} = \rho_i + t \tag{24}$$

While their angle of distribution is

$$\theta_i = sin^{-1}\left(\frac{L_{ie}/2}{\rho_i}\right) \tag{25}$$

Where *t* is leaf spring thickness and it is constant .

Equations (24) and (25) are the geometrical representation of the leaf spring in polar coordinate which take into account the shape effect in terms of L_e, which is calculated according to equation (8) for trapezoidal shape and according to equation (12) for circular shape, and the curvature distribution in term of radius of curvature (ρ) and angle of distribution (θ). **Table. 2** and **Table. 3** indicates the dimensions of circular and parabola leaf springs.

Table 2. Geometrical representation of trapezoidal shape – circular curvature leaf spring in polar coordinate.

Leaf No.	L_e	ρ	θ
1	900	606.25	47.92
2	700	616.25	34.607
3	500	626.25	23.53
4	300	636.25	13.636
5	100	636.25	4.434

Table. 3 Geometrical representation of circular shape – circular curvature leaf spring in polar coordinate.

Leaf No.	L_e	ρ	θ
1	993.29	716.64	43.87
2	952.014	726.64	40.93
3	863.44	736.64	35.889
4	709.48	746.64	28.375
5	408.752	756.64	15.675

4- Finite Element Analysis:

Due to its thinness, leaf spring base plate has been analytically dealt with as a straight plate regarding its induced bending stress calculation no matter what type of curvature it has been made of, such assumption leads to some errors which is related to the curved beam shape complications that are radius of curvature calculation and the amount of eccentricity, such divergence will be numerically studied using FEM with the aim of investigating the best leaf spring curvature and shape with respect of increasing its ***load carrying capacity*** as well as ***enhancing its dynamic performance*** regarding ***fatigue resistance*** and ***vibration reduction***. The studied cases have been investigated as ***simply supported leaves (S_y= S_z=0 at***

its end lower edges that are freely move in x direction) working under ***surface to surface contact as a smooth nonlinear behavior that is solved depending on Lagrange Penelty algorithm*** and ***bending mechanisms which are simulated under static and completely reversed dynamic loads*** with its ***maximum value of 2000 N*** using Ansys Workbench software ver.15. Previous works have referred that the leaf spring central bolt existence and the effect of the leafs internal friction and the shackle and bush could be eliminated during the investigation process because they complicate the problem with a dispensable change in the investigation results [1],[11]. The investigated material has been chosen to follow Hook's Low and designed to be solved according to ***Soderberge*** theory regarding ***fatigue life and safety factor estimation*** which have been solved automatically using FEM environment. The mechanical properties of the studied cases are listed in **Table.1** and the S-N behavior is shown in **Fig.5**. The optimum element types and number has been reached depending on the convergence test as shown in **Fig.6** and found to be ***brick element 20 node 186*** with fine size.

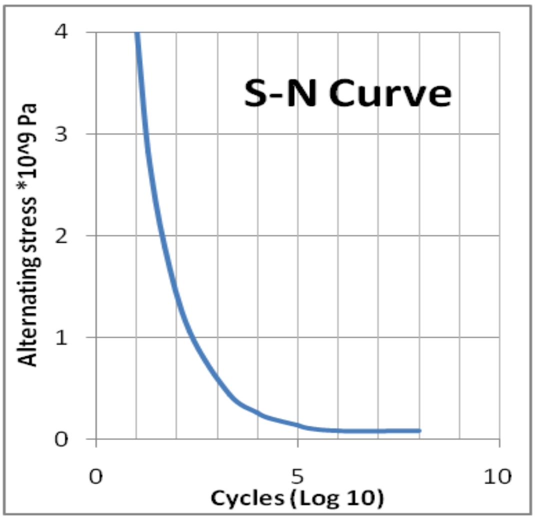


Fig. 5 Investigated leaf spring S- N curve

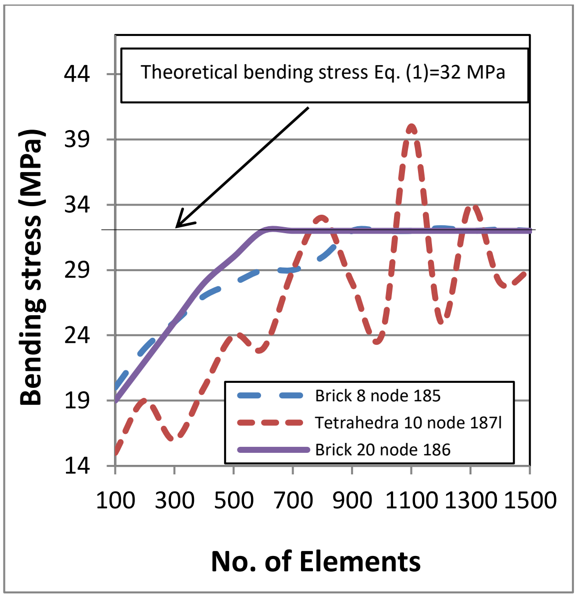


Fig. 6 Convergence test diagram

5- Results and Discussions:

With the aim of increasing the leaf spring load carrying capacity, this work proposed the circular plate shape as the base to build up the leaf spring instead of the trapezoidal one and introduced an additional two curvatures along with the traditional circular curvature that are parabola curvature and exponential one. The strength and modal results indicates some facts to be discussed and explained in detail.

The validity of problem representation regarding the geometrical representation as well as the boundary condition and applied load status reality have been verified depending on equation (1) showing a very good agreement proving that the investigation could be extended to cover the whole study numerically. Using equation (1) depending on the data in **Table. 1** yields that the ***bending stress = 30 MN/m^2*** while the ***numerical results in*** **Fig. 7** ***is 32 MN/ m^2*** showing that the ***discrepancy percentage is about*** *6%*. From the numerical results **Fig.8** shows the analytical and numerical variation of maximum bending stresses with the variation of the applied load.

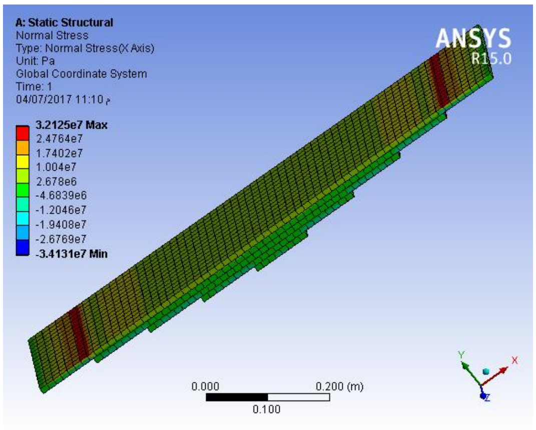


Fig.7 Trapezoidal shape- straight leaf spring bending stress

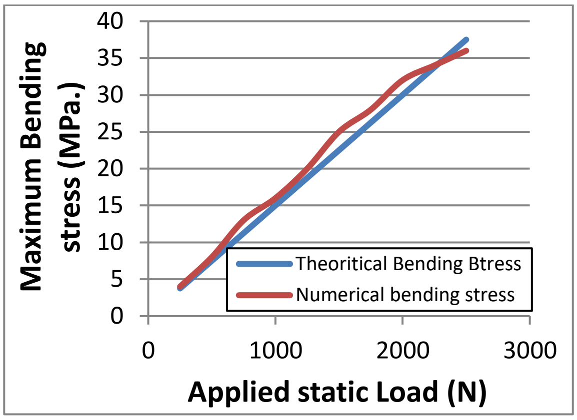


Fig.8 Maximum bending stress variation with the variation of applied load for trapezoidal shape - straight leaf spring

from the whole results the following facts could be noticed

1. It has been found that the static and dynamic performance for proposed circular shape leaf spring is better than its counterpart trapezoidal shape one for all of the curvature. see **Table. 4** and **Table. 5**.
2. Shape effect: for circular curvature, the proposed circular shape leaf spring reduces the bending stress by about 35.6% compared with the trapezoidal shape see **Fig. 9-a** and **Fig. 10-a**.

Table.4 Static stress results for different shapes and curvatures combinations.

curvature	Bending stress MN/m^2	
	Trapezoidal shape	Circular shape
Circular	11.041	7.1147
Parabola	18.833	7.7451
Exponential	31.424	7.2085

Table.5. Safety factor results for different shapes and curvatures combinations under completely reversed dynamic load.

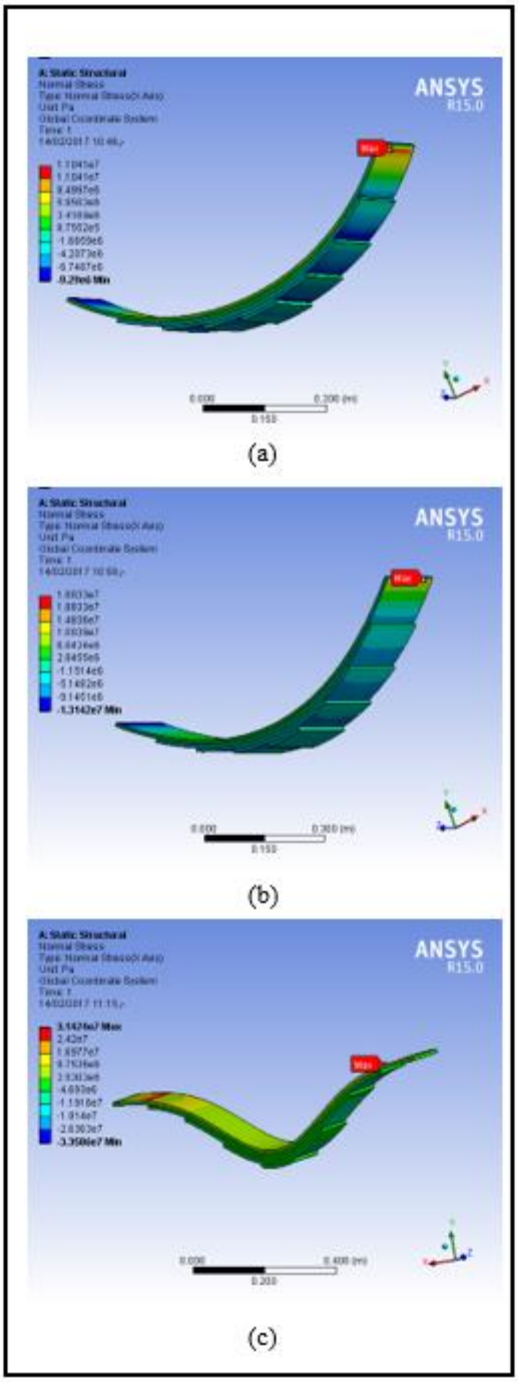
Leaf spring curvature	Safety factor	
	Trapezoidal	Circular
Circular	3.2428	5.0275
Parabola	2.1023	4.7938
Exponential	2.6667	12.446

3. curvature effect: in general, the alternative parabola and exponential curvature are badly affect the load carrying capacity for trapezoidal shape leaf spring with obvious increasing in the bending stress by about 63% for exponential curvature and by about 43% for parabola one see **Fig. 9-a,b** and **c** with worthless effect for circular shape see **Fig. 10-a,b** and **c**.
4. Dynamic performance: using the exponential curvature has the dominant role regarding the durable dynamic performance that increasing the safety factor by about 300%, see **Fig. 11-a** and **Fig.12-c**.
5. The natural frequency of the exponential leaf spring is always lower than the other two leaf spring curvatures leading to larger vibrating amplitudes according to equation. (3), see **Table. 6**
6. The circular plate shape increases the leaf natural frequencies and play a positive role in free vibration performance, where the greater natural frequency of the leaf spring leads to enhances the impact damper functions [2].

All of the previous results could be attributed to the effect of leaf spring stiffness changes which is affected by the leaf spring curvature and shape in addition to that the use of curved beam decomposes the applied load into two components parallel and perpendicular to the curvature at any point leading to induce bending and tensile stress components instead of the bending stress only in case of straight beam, so that the curvature redirect the applied load at any section rather than the central section leading to change the induced stresses as well as its contribution in the stiffness change

Table.6. Fundamental natural frequency for different leaf spring shapes and curvatures

Leaf spring curvature	Frequency (Hz)	
	Trapezoidal shape	Circular shape
Circular	89.3	104.5
Parabola	76.8	105.4
Exponential	47.5	84



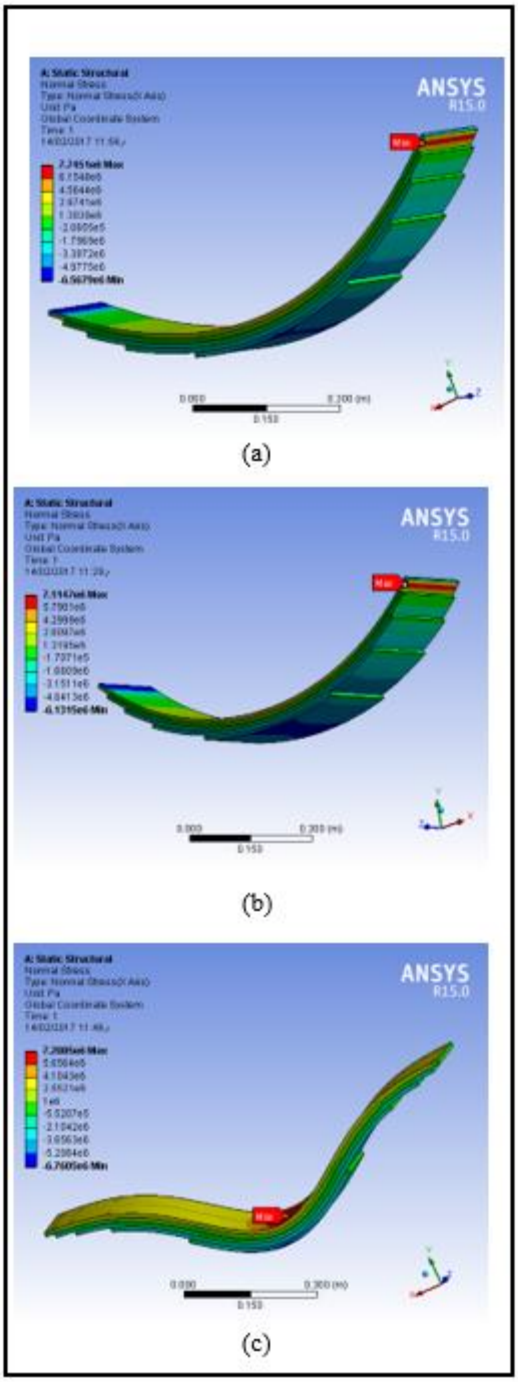


Fig. 9 Bending stress of trapezoidal leaf spring shape for three proposed curvatures a)circular. b) parabola. c) exponential

Fig. 10 Bending stress of circular leaf spring shape for three proposed curvatures a)circular. b) parabola. c) exponential

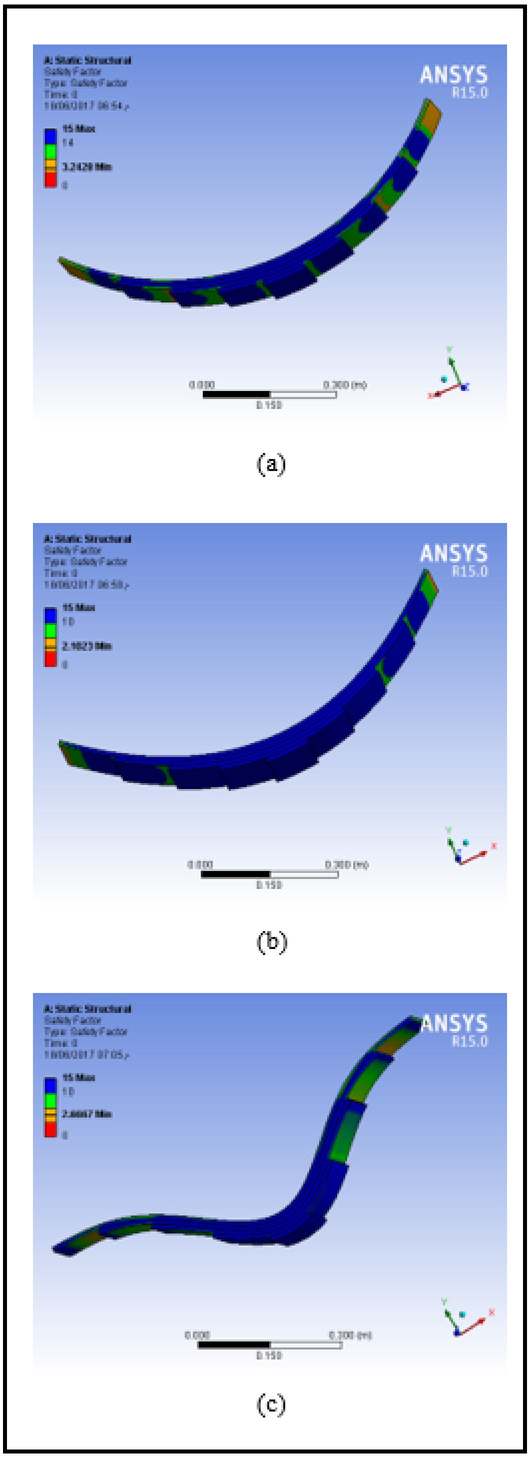


Fig.11 Safety factor for trapezoidal leaf spring shape accompanied by the three proposed curvatures a) Circular. b) Parabola. c) Exponential

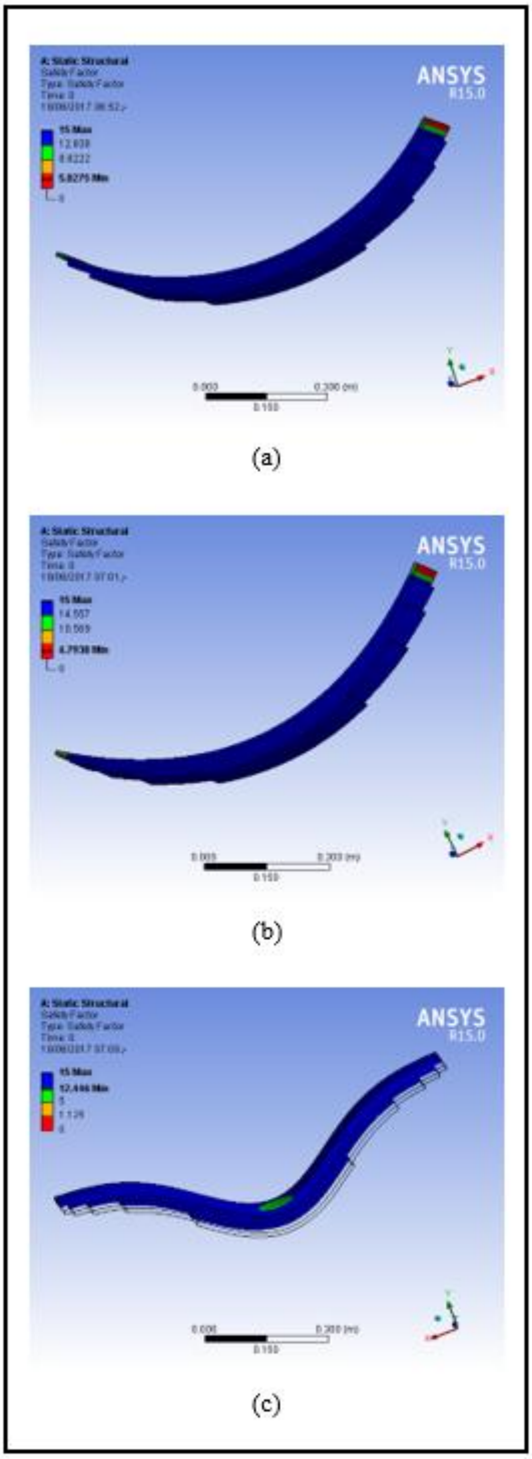


Fig.12 Safety factor for circular leaf spring shape accompanied by the three

proposed curvatures a) Circular. b) Parabola. c) Exponential

6- Conclusions:

Depending on the whole results of this work according with its assumption and working conditions there some conclusions to be summarized as

1. Adopting the circular shape leaf spring is better than the trapezoidal one regarding the leaf carrying capacity enhancement.
2. The alternative exponential curvature accompanied by the circular shape leaf spring has the key role regarding fatigue resistance about 300% enhancement on the safety factor
3. Using exponential curvature leaf spring must be associated with an efficient damping system to overcome its large vibration amplitude.

7. Nomenclature:

A: area, mm^2
b: width of the plate, mm
B: base plate width, mm
E: Modulus of elasticity, N/mm^2
L: base plate Length, mm.
L_i: individual original leaf strip length, mm.
L_{ie}: equivalent rectangular leaf strip length, mm
n: Number of leaves.
S_y,S_z: displacement in y and z directions respectively, mm
t: Thickness of plate, mm
W:Applied load, N
$\sigma_{b:}$ bending stress, N/mm^2
$\sigma_{e:}$ endurance limit, N/mm^2.
ρ: Radius of curvature, mm
δ:leaf spring initial deflection, mm
δ_{max}: maximum static bending deflection, mm
δ_{Dc} : Critical vibration amplitude, mm

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تأثير منحني و شكل النابض الورقي على سعة أداة السكوني والديناميكي

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الخلاصة:

لم تعرف ظروف الترفيه في وسائط النقل قبل ظهور النابض والمخمد كأجزاء مدمجة في أنظمة التعليق. تأخذ أشكال النوابض هيئات عدة إعتماداً على واجباتها و ظروف العمل بالإضافة الى المجال المتوفر.نتيجة قابليتها لإمتصاص مقدار كبير من التشوه, أُعتُمِدت آلية حناية العتبات في أنظمة تعليق سيارات الحمل. يقترح هذا العمل نوابض ورقية متعددة الطبقات مصنعة من صفيحة دائرية الشكل توزع طبقاً لمجموعة من المنحنيات المختلفة (دائرية, قطع مكافيء, أسية للأساس الطبيعي) كتعديلات هندسية لزيادة سعة نقل الأحمال بتقليل إجهادات الحناية المتولدة ومحسناً عمر الكلال او معامل الأمان الديناميكي وتحري تأثيراتها على خواص الاهتزاز. تم إعتماد المنحنيات المقترحة لتكون نوابض ورقية بديلة للنوابض التقليدية ذات الشكل المعيني دائري الحناية وتم تمثيلها رياضياً بالإحداثيات القطبية وتغذيتها ليتم محاكاتها وحلها عددياً بإستخدام برنامج Ansys Workbench الاصدار الخامس عشر. اهم نتائج هذا العمل هي ان الشكل الدائري المقترح للّوحة الاساسية للنابض الورقي تقلل اجهاد الحناية السكوني الاعظم بحوالي 36% بغض النظر عن نوع حنايته وتم تحسين معامل الامان الديناميكي تحت الاحمال الترددية المعكوسة كلياً بحوالي 300% بأستخدام منحني الاساس الطبيعي بدلا من المنحني الدائري التقليدي.

الكلمات المفتاحية: النوابض الورقية متعددة الطبقات, شكل ومنحني النابض الورقي, اجهادات النابض الورقي