Performance Study for Mixed Transforms Generated by Tensor Product in Image Compression and Processing

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Abstract:-

In all applications and specially in real time applications, image processing and compression plays in modern life a very important part in both storage and transmission over internet for example, but finding orthogonal matrices as a filter or transform in different sizes is very complex and importance to using in different applications like image processing and communications systems, at present, new method to find orthogonal matrices as transform filter then used for Mixed Transforms Generated by using a technique so-called Tensor Product based for Data Processing, these techniques are developed and utilized. Our aims at this paper are to evaluate and analyze this new mixed technique in Image Compression using the Discrete Wavelet Transform and Slantlet Transform both as 2D matrix but mixed by Tensor Product. The performance parameters such as Compression Ratio, Peak Signal to Noise Ratio, and Root Mean Squared Error, are all evaluated for both standard colored and gray images. The simulation result shows that the techniques provide the quality of the images it was normal but acceptable and need more researchers works.

Keywords—Mixed Transforms, Tensor Operation, Orthogonal Matrices, Wavelet Transform, Slantlet Transform, Images Compression.

1. Introduction To New Mixed Transforms

In this research work meaning of the mixing transformer is transfer signal from domain to another spatial domain by using multiple orthogonal filters transform for collect the benefits and properties of all converters transforms in the same signal, in our case the signal will be 2D image both color and gray scale. In the previous case or old classic methods techniques for the mixing process will happing by the form of stages as presented in **Fig. 1**, this segment will take several cases to find Mixed Transforms from two or more well - known Transforms, the first Wavelets and its Groups (DWT Discrete Wavelet Transform), and the second one the Slantlet Transform as 2D matrix procedure.

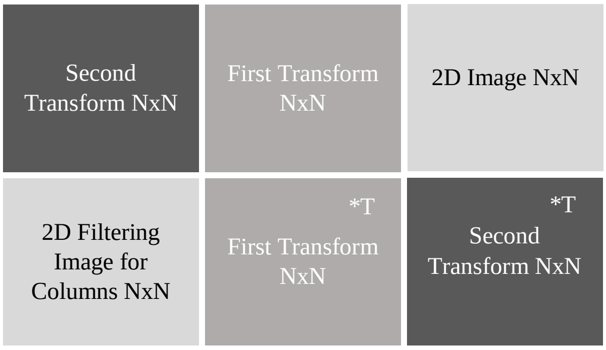


Fig. 1 Mix Transforms Algorithm for 2D Images (Stander method).

It can be creating a Mixed Transforms from the above transformer as following:

1. 2D Wavelet (DWT Discrete Wavelet Transform), in deferent sizes with Wavelet its self in deferent sizes too,

2. 2D matrix Slantlet Transform in deferent sizes with Slantlet also in deferent sizes,

At the end there are many for mix two transforms or more the reason for this way to achieve the benefit for this transforms in one matrix or different sizes of filters to different dimensions of objects and study the performances, "it is very importance think that the first matrix will be called the base or the root and the other the branches or leaves in illustration section 1, we find that the mix matrix is take the figure of base matrix if we can mix the Wavelet possessions and Fourier properties we can get the frequency domain and Spatial domain, together without using old ways as shown (as in **Fig. 1**) for images as example application it will be desiccation in the next section"[6].

For the new method, will be custom the tensor product, may be valuable in dissimilar ways in vectors, matrices, spaces, algebras, topological vector, and modules, among several add-on structures or objects. "The most general multi-linear process, in some contexts, this product is also cited to as outer product. Matrices can denote the linear maps A and B. Then, the matrix relating the tensor product is the ***Kronecker multiply*** of the two or more matrices. There are many properties for tensor product but for this paper we will impervious a new property in this paper can be functional for our investigation and the property is" [6]:

Theorem 1 If **A:** $Z \rightarrow Z$ and **B:** $Z \rightarrow Z$ are square matrices, and both A and B are orthogonal matrices then any act of their tensor product on a matrix is given by ($A \otimes B = C: Z \rightarrow Z$) the matrix **C** will be square orthogonal as well, somewhere **Z** is the set for complex numbers together with the vectors numbers [6] .

This theorem can be practical even on vector number with vector or matrix numbers but must both orthogonal, the proof in [6] well help to understanding this new property as shown in Example:

Let Wavelet Db2 2x2 $[\mathbf{A}] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ and

Wavelet Db2 4x4 $[\mathbf{B}] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

then:

$$[\mathbf{A}]_{2x2} \otimes [\mathbf{B}]_{4x4} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & -1 \\ 1 & -1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Then to proof the orthogonally for matrix **C** it is must make matrix multiply:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & -1 \\ 1 & -1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 1 & -1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & -1 \end{bmatrix} = 4[\mathbf{I}]_{8x8}$$

2. IMAGE COMPRESSION USING MIX TRANSFORM

The image compression is a data compression procedure of numerical images. The goal of the digital image looseness is to minimalize the idleness of the image data in order to transfer and stock data in the effective system [6], uphold the energy retained and growth the compression ratio. A technique for the image compression is explained; in the Wavelet Transform the coefficients below a certain value of threshold are removed therefore a total threshold can be used to progress Wavelet compression method [3]. There are several of image compression parameters, some of them are:

1. Energy: The energy is a measurement of pixel pair's repetitions. It can be figured by:

$$\text{Energy} = \sum\sum P^2(i,j) \qquad (1)$$

where in equ. (1) $P_{(i,\ j)}$ is the probability density here [5].

2. Peak Signal to Noise Ratio (**PSNR**): To quantify the image quality and to measure the similarity between the original image and reconstructed image, PSNR parameter can be used. A better image quality can be produced by larger PSNR [4]. It can be calculated by [1]:

$$\text{PSNR} = 10 \log_{10}\left(\frac{255^2}{\text{MSE}}\right) \quad (\text{dB}) \qquad (2)$$

where (MSE) is mean square error

between any two images which is given by the equation:

$$MSE = \frac{1}{k}\sum_{i=1}^{k}(P_i - Q_i)^2 \quad (3)$$

where P_i is the original image, Q_i is the compressed image data and "k" size of the image.

3. Root Mean Square Error (RMSE): It is the square root of a mean square error [2]:

$$RMSE = \sqrt{MSE} \quad (4)$$

From calculations above, final equations will be:

1. The transform construction equation:

$$[\mathbf{T}]_{2^m x 2^m} = [\mathbf{WD2}]_{2^n x 2^n} \otimes [\mathbf{WD2}]_{2^{m-n} x 2^{m-n}} \quad (5)$$

where: n variation from 1 to m-1 for our example in this paper as shown in tables from 1 to 4 we take 2048x2048 or $2^{11}x2^{11}$ the size of the original Gray Image, and take DWT as following from equation (5):

$[\mathbf{T}]_{2^{11} x 2^{11}} = [\mathbf{WD2}]_{2^1 x 2^1} \otimes [\mathbf{WD2}]_{2^{10} x 2^{10}}$
then:

$[\mathbf{T}]_{2^{11} x 2^{11}} = [\mathbf{WD2}]_{2^2 x 2^2} \otimes [\mathbf{WD2}]_{2^9 x 2^9}$ then:

$[\mathbf{T}]_{2^{11} x 2^{11}} = [\mathbf{WD2}]_{2^3 x 2^3} \otimes [\mathbf{WD2}]_{2^8 x 2^8}$ then:

:

$[\mathbf{T}]_{2^{11} x 2^{11}} = [\mathbf{WD2}]_{2^{10} x 2^{10}} \otimes [\mathbf{WD2}]_{2^1 x 2^1}$

Note that For Slantlet Transform is the same for DWT and for Color the image size will be $2^{10}x2^{10}$.

2. the original image will be three parts Red, Green and Blue:

$$[\mathbf{Img}] = [\mathbf{P}]_{2^m x 2^m \boldsymbol{R}}, [\mathbf{P}]_{2^m x 2^m \boldsymbol{G}}, [\mathbf{P}]_{2^m x 2^m \boldsymbol{B}}$$

3. find the result for the equations:

$$[\mathbf{Q}]_{2^m x 2^m} = [\mathbf{T}][\mathbf{Img}][\mathbf{T}]^{*\mathbf{T}}$$
$$= [\mathbf{T}][\mathbf{P}]_{2^m x 2^m \boldsymbol{R}}, [\mathbf{T}][\mathbf{P}]_{2^m x 2^m \boldsymbol{G}}[\mathbf{T}]^{*\mathbf{T}}, [\mathbf{T}]\ [\mathbf{P}]_{2^m x 2^m \boldsymbol{B}}[\mathbf{T}]^{*\mathbf{T}} \quad (6)$$

4. take the first quarter or $[\mathbf{Q}]_{2^{m/2} x 2^{m/2}}$ then now can be transmitter or store.

5. for reconstruction the image $[\mathbf{G}]_{2^m x 2^m} = [\mathbf{Q}]_{2^{m/2} x 2^{m/2}} + [\mathbf{Zero}]_{2^m x 2^m}$ **where** $[\mathbf{Zero}]_{2^m x 2^m}$ **is Zeros matrix.**

6. then the final image will be

$$[\mathbf{Img}]_{2^m x 2^m} = [\mathbf{T}]^{*\mathbf{T}}[\mathbf{G}]_{2^m x 2^m}[\mathbf{T}]$$
$$= [\mathbf{T}]^{*\mathbf{T}}[\mathbf{G}]_{2^m x 2^m \boldsymbol{R}}[\mathbf{T}], [\mathbf{T}]^{*\mathbf{T}}[\mathbf{G}]_{2^m x 2^m \boldsymbol{G}}[\mathbf{T}], [\mathbf{T}]^{*\mathbf{T}}\ [\mathbf{G}]_{2^m x 2^m \boldsymbol{B}}[\mathbf{T}] \quad (7)$$

7. Peak Signal to Noise Ratio (**PSNR**) will be:

$$MSE = \frac{1}{k}\sum_{i=1}^{k}(P_i - Img_i)^2 \quad (8)$$

And Root Mean Square Error $RMSE = \sqrt{MSE}$

All result for the above equations can be study well in the **Tables** from **1** to**4**:

Table. 1 The Results of (Wavelet) Tensor Product

1st Matrix Size (M1)	2nd Matrix Size (M2)	No# Of Sub-Band Of Mixed Image	Energy of LL Sub-Band	PSNR	RMSE
2×2	1024×1024	2×2	1.7738	33.9938	5.1115
4×4	512×512	4×4	2.5387	35.0936	4.5036
8×8	256×256	8×8	1.6405	41.1319	2.2472
16×16	128×128	16×16	1.1741	48.4519	0.9675
32×32	64×64	32×32	1.6829	48.6761	0.9428
64×64	32×32	64×64	1.5682	55.5744	0.5183
128×128	16×16	128×128	1.4877	58.8925	0.3497
256×256	8×8	256×256	1.4824	65.8984	0.1298
512×512	4×4	512×512	1.5185	70.6603	0.0750
1024×1024	2×2	1024×1024	1.5110	84.9407	0.0145

Table. 2 The Results of (Slantlet) Tensor Product for Gray Image

1st Matrix Size (M1)	2nd Matrix Size (M2)	No# of Sub-Band Of Mixed Image	"Red" LL Sub-Band Energy	"Green" LL Sub-Band Energy	"Blue" LL Sub-Band Energy	PSNR of "Red" Band	PSNR of "Green" Band	PSNR of "Blue" Band	RMSE of "Red" Band	RMSE of "Green" Band	RMSE of "Blue" Band
2×2	512×512	2×2	0.7006	0.2104	0.1096	43.8275	41.8245	41.6127	2.1172	2.0750	2.1262
4×4	256×256	4×4	0.4929	0.1357	0.0475	39.7548	42.8426	44.3864	3.4772	2.3858	1.5450
8×8	128×128	8×8	0.8465	0.1889	0.0084	42.9081	45.9646	43.4502	2.3669	1.6353	1.7208
16×16	64×64	16×16	0.9099	0.1679	0.0728	46.4676	46.1341	50.2651	1.5391	1.6022	0.7852
32×32	32×32	32×32	0.7275	0.2104	0.0278	57.1958	56.4455	50.8632	0.4276	0.4674	0.7330
64×64	16×16	64×64	0.6104	0.2202	0.0337	60.4681	54.5466	51.2610	0.2426	0.4796	0.7001
128×128	8×8	128×128	0.7608	0.2320	0.0530	59.3510	54.5118	52.8010	0.2759	0.4816	0.5864
256×256	4×4	256×256	0.6932	0.2134	0.0240	53.9698	51.3522	51.2047	0.5126	0.6928	0.7047
512×512	2×2	512×512	0.7152	0.2295	0.0214	53.9369	51.3784	51.2331	0.5145	0.6907	0.7024

Table. 3 The Results of (Wavelet) Tensor Product for Color Image

1st Matrix Size (M1)	2nd Matrix Size (M2)	No# of Sub-Band Of Mixed Image	"Red" LL Sub-Band Energy	"Green" LL Sub-Band Energy	"Blue" LL Sub-Band Energy	PSNR of "Red" Band	PSNR of "Green" Band	PSNR of "Blue" Band	RMSE of "Red" Band	RMSE of "Green" Band	RMSE f "Blue" Band
2×2	512×512	2×2	0.4346	0.0254	0.0090	43.8275	41.8245	41.6127	2.1172	2.0750	2.1262
4×4	256×256	4×4	0.5819	0.0680	0.0346	37.6148	41.1033	50.6396	4.5246	2.9485	0.9330
8×8	128×128	8×8	1.1174	0.1498	0.0025	41.4094	44.8595	43.9096	2.8404	1.8686	1.6322
16×16	64×64	16×16	1.3332	0.5127	0.1848	58.9588	48.3333	50.0048	0.3470	1.2299	0.8091
32×32	32×32	32×32	1.3048	0.3317	0.1624	57.3495	59.3156	53.2305	0.3473	0.3326	0.5581
64×64	16×16	64×64	1.5086	0.1749	0.1138	61.8882	56.4204	53.3412	0.2455	0.3866	0.5510
128×128	8×8	128×128	1.7231	0.3172	0.1319	66.4304	57.0376	55.4054	0.1221	0.3600	0.4345
256×256	4×4	256×256	1.3866	0.2115	0.0771	56.9824	54.1930	53.8474	0.3623	0.4996	0.5198
512×512	2×2	512×512	1.3076	0.2090	0.0756	54.7462	53.0308	53.2306	0.4687	0.5711	0.5581

Table. 4 The Results of (Slantlet) Tensor Product for Color Image

1st Matrix Size (M1)	2nd Matrix Size (M2)	No# Of Sub-Band Of Mixed Image	Energy of LL Sub-Band	PSNR	RMSE
2×2	1024×1024	2×2	1.6627	33.9938	5.1115
4×4	512×512	4×4	3.8200	35.2405	4.4281
8×8	256×256	8×8	2.9560	39.5438	2.6981
16×16	128×128	16×16	2.0213	40.2806	2.4786
32×32	64×64	32×32	2.0697	47.3442	1.0991
64×64	32×32	64×64	2.2290	56.3576	0.3894
128×128	16×16	128×128	2.1896	83.6799	0.0168
256×256	8×8	256×256	2.1232	66.3734	0.1229
512×512	4×4	512×512	2.0869	73.9637	0.0594
1024×1024	2×2	1024×1024	2.0807	74.7335	0.0469

3. RESULTS AND CONCLUSION

From Fig. 2 to Fig. 9 will be shown that it can to make study and compare between the two transform DWT and Slantlet just in this paper, in Image Compression, but also, it can make long study for all types of transform and for combine two or more transforms like Wavelet and Slantlet with Fourier. In the end in this paper and the proposal method, the conclusion for this result in tables from 1to 4 and from **Fig. 2** to **Fig. 9** can be summarize as fallowing:

• it is obviously strong that, it can be realized in the first, the new method is not all case can be suitable for Image Comparisons as showed except the most obvious case when the transform for both DWT and Slantlet and both in Color and Gray, as fallowing:

$$[\mathbf{T}]_{2^m x 2^m} = [\mathbf{WD2}]_{2^1 x 2^1} \otimes [\mathbf{WD2}]_{2^{m-1} x 2^{m-1}}$$

or

$$[\mathbf{T}]_{2^m x 2^m} = [\mathbf{SLT}]_{2^1 x 2^1} \otimes [\mathbf{SLT}]_{2^{m-1} x 2^{m-1}}$$

• it is obviously strong that, this new technique can work as encryptions tools and comparisons in the same time if the transform will work as encryptions private key.

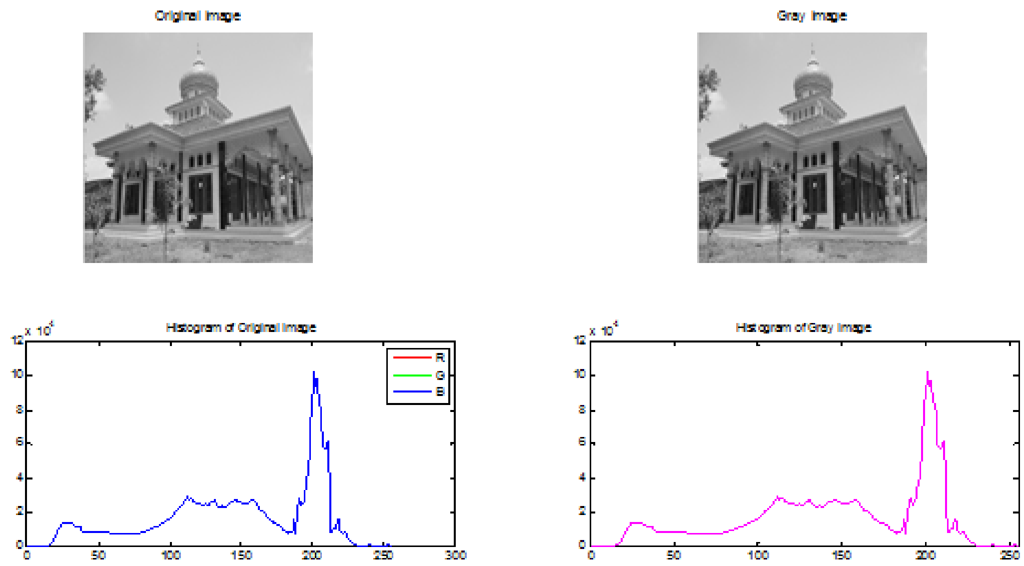


Fig. 2 Gray image: The model of (wavelet) tensor product for best result

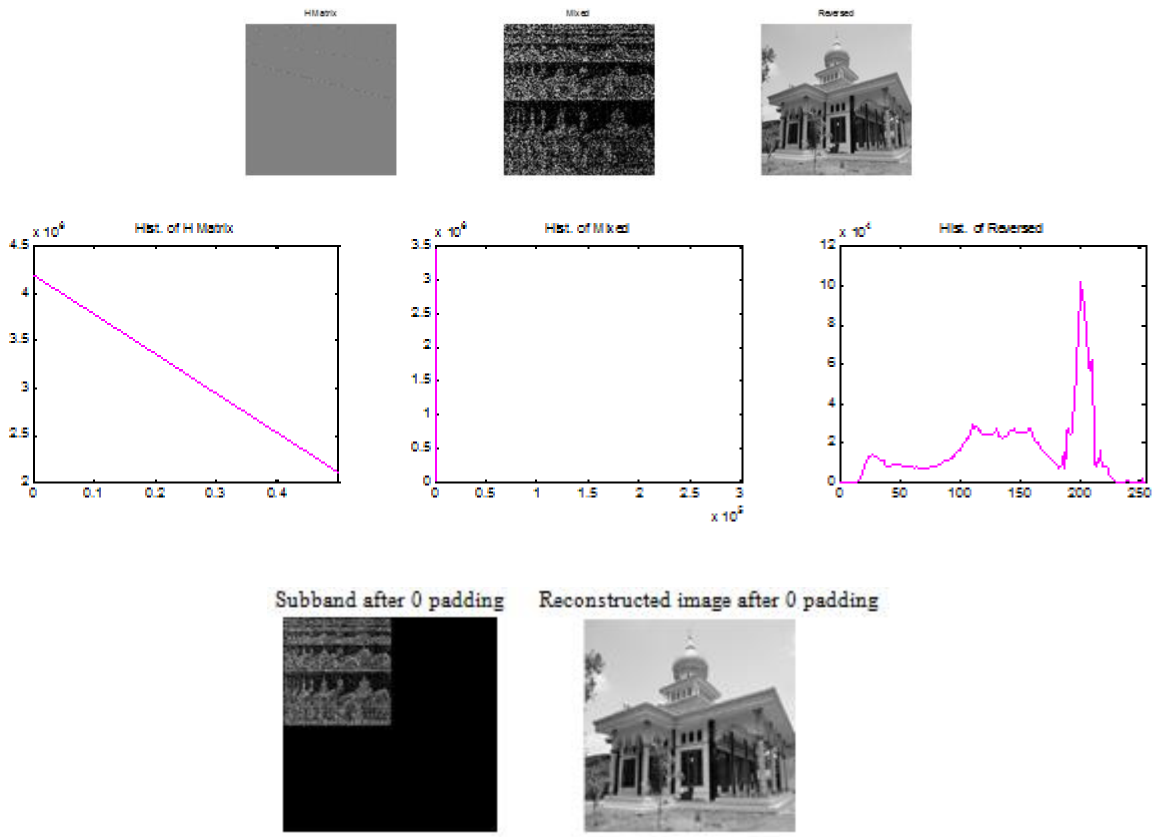


Fig. 3 Gray image: The model of (wavelet) tenser product after Mix.

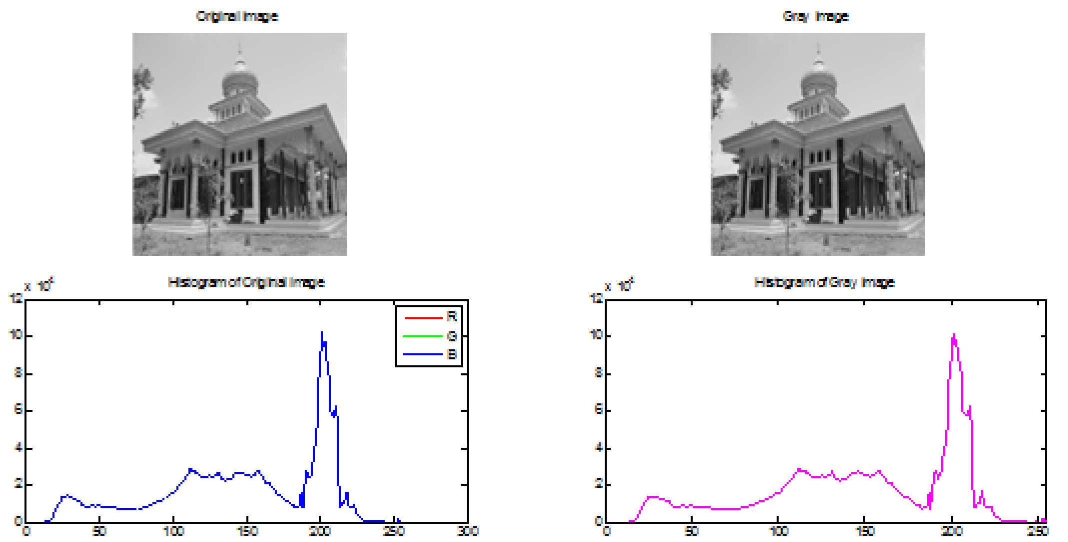


Fig. 4 Gray image: The model of (Slantlet) tenser product for best result

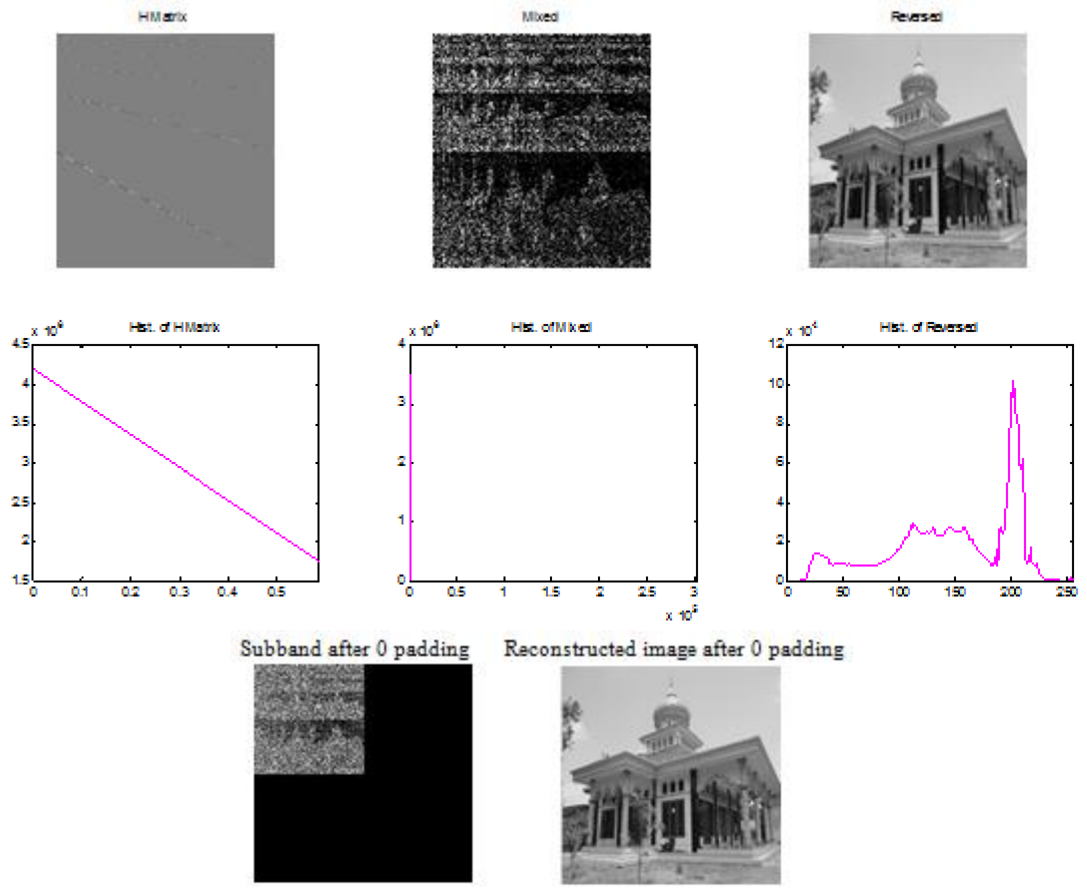


Fig. 5 Gray image: The model of (Slantlet) tenser product after Mix. and reconstruction.

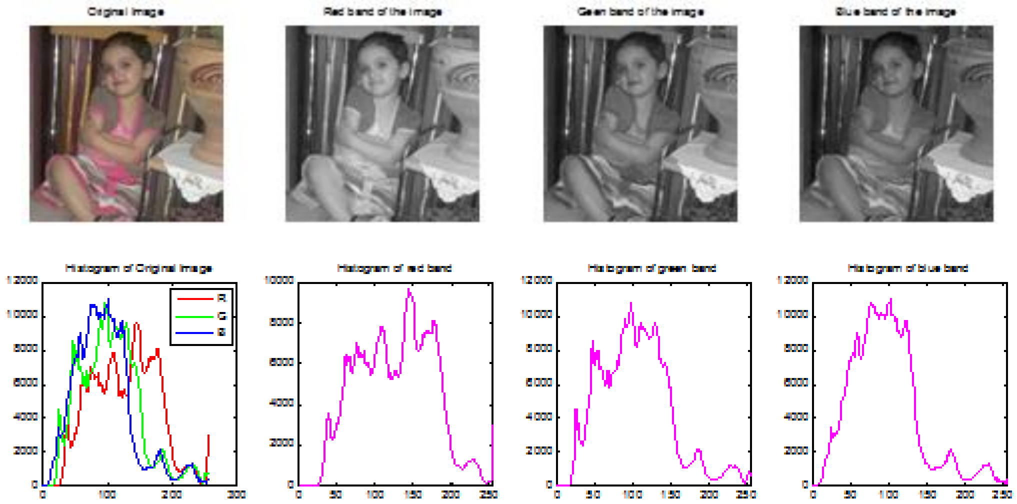
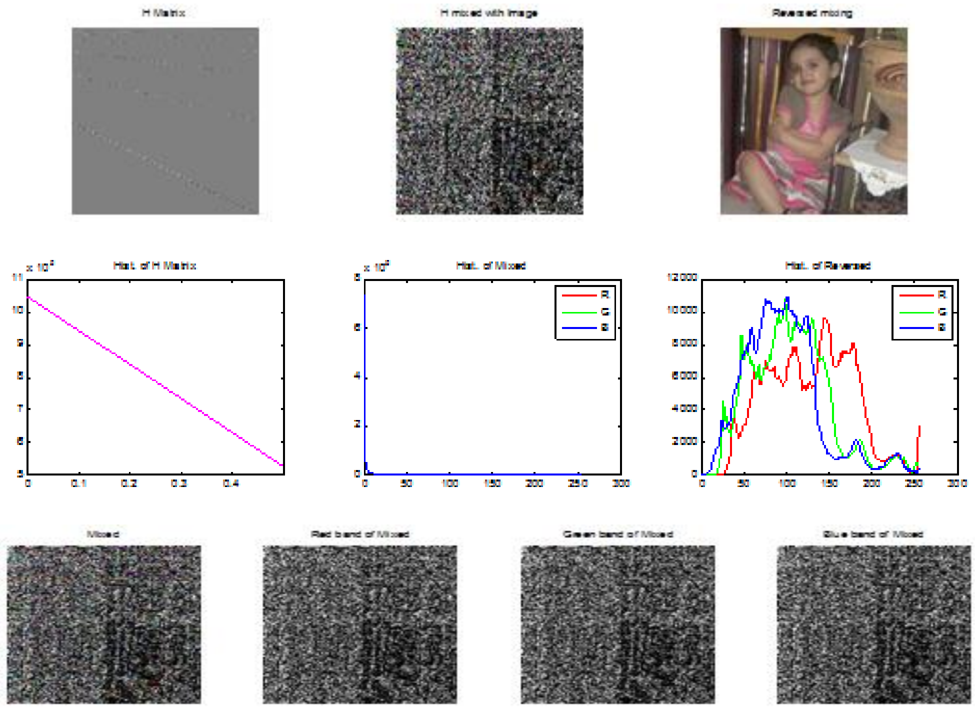


Fig. 6 Coloer image: The model of (wavelet) tenser product for best result



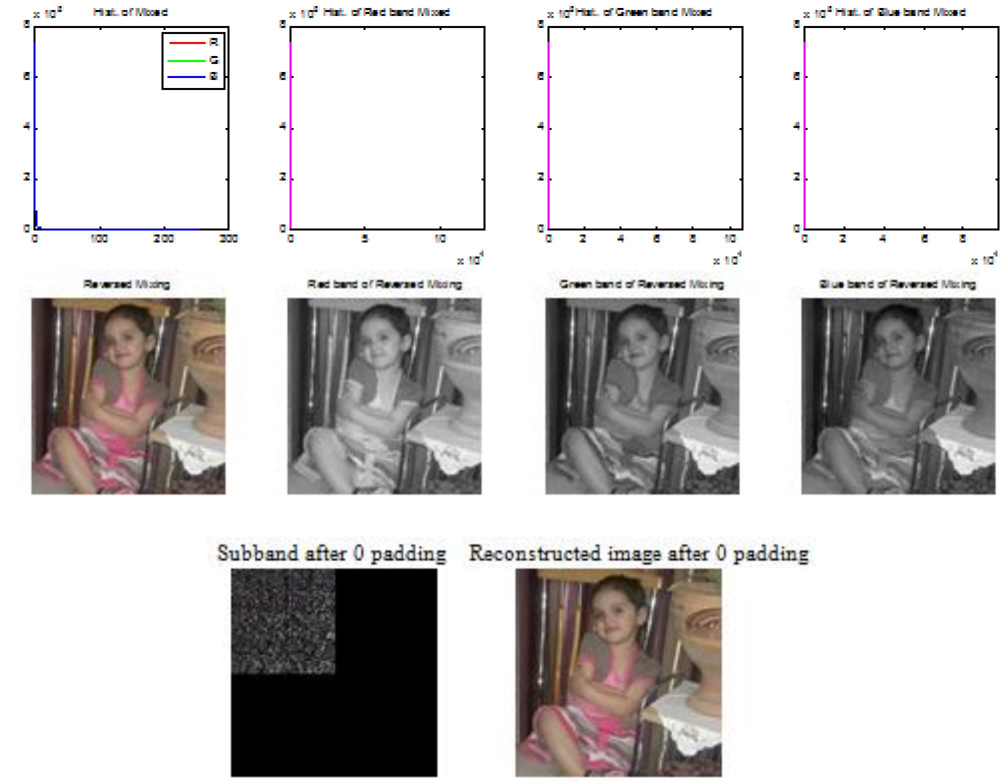


Fig. 7 Color image: The model of (Wavelet) tenser product after Mix. and reconstruction.

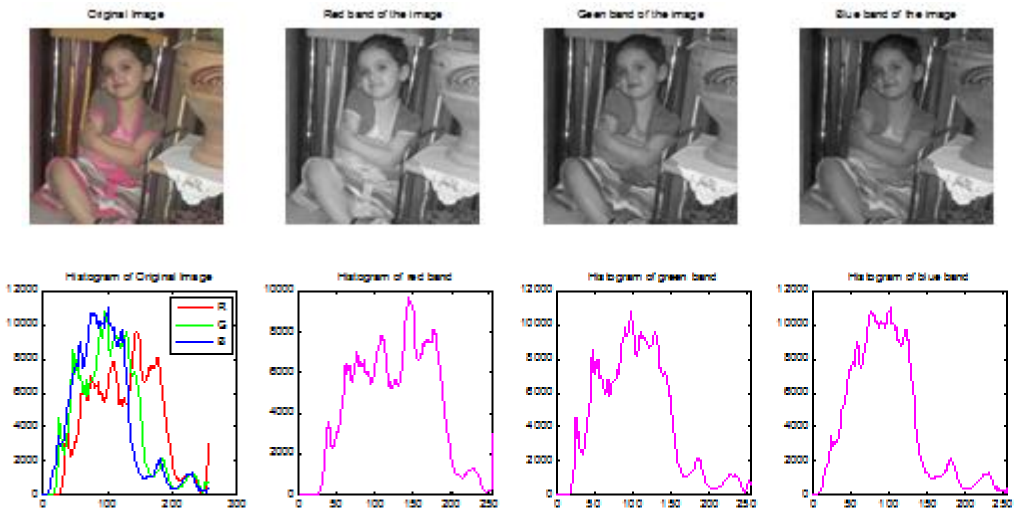


Fig. 8 Color image: The model of (Slantlet) tenser product for best result.

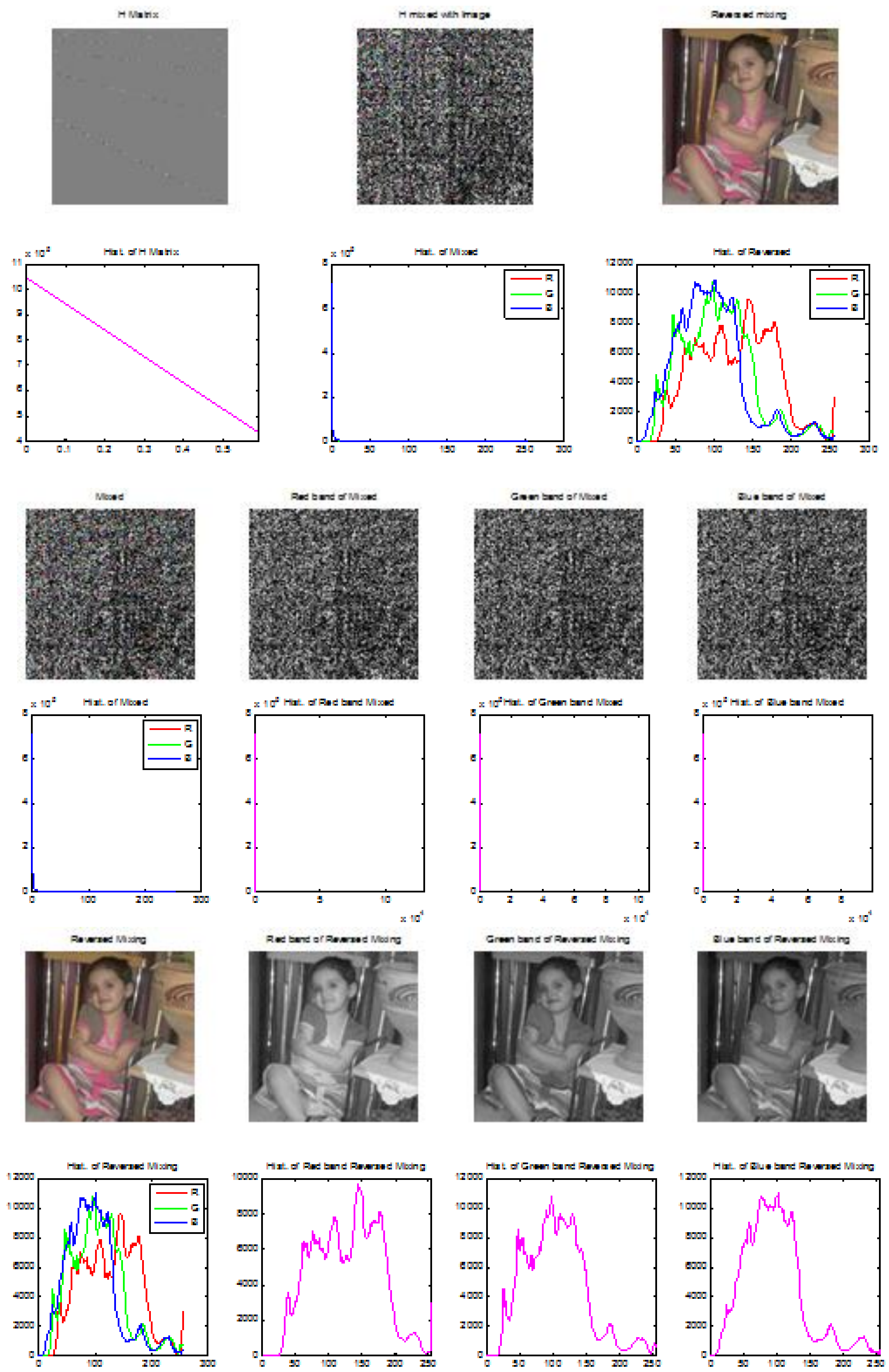

H Matrix
H mixed with image
Reversed mixing
Hist. of H Matrix
Hist. of Mixed
Hist. of Reversed
R
G
B
Mixed
Red band of Mixed
Green band of Mixed
Blue band of Mixed
Hist. of Mixed
Hist. of Red band Mixed
Hist. of Green band Mixed
Hist. of Blue band Mixed
Reversed Mixing
Red band of Reversed Mixing
Green band of Reversed Mixing
Blue band of Reversed Mixing
Hist. of Reversed Mixing
Hist. of Red band Reversed Mixing
Hist. of Green band Reversed Mixing
Hist. of Blue band Reversed Mixing

Fig. 9 Color image: The model of (Slantlet) tenser product after Mix. and reconstruction.

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دراسة الأداء للتحويلات المختلطة الناتجة عن منتج الموتر في ضغط الصور ومعالجتها

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الخلاصة:-

في جميع التطبيقات وخاصة في التطبيقات في الوقت الحقيقي، ومعالجة الصور والضغط والذي يلعب حاليا في الحياة الحديثة جزءا هاما جدا في كل من التخزين والنقل عبر الإنترنت على سبيل المثال، ولكن العثور على المصفوفات المتعامدة وايضا مرشح أو تحويل في أحجام مختلفة معقد جدا و أهمية لاستخدامها في تطبيقات مختلفة مثل معالجة الصور ونظم الاتصالات، وفي الوقت الحاضر، تم اكتشاف طريقة جديدة للعثور على المصفوفات المتعامدة ومنها مرشح التحويل ثم تستخدم للتحويلات المختلطة التي تم إنشاؤها باستخدام تقنية ما يسمى المنتج الموتر على أساس معالجة البيانات، يتم استخدام وتطوير هذه التقنيات. أهدافنا في هذه الورقة هي لتقييم وتحليل هذه التقنية المختلطة الجديدة في ضغط الصورة باستخدام تحويل المويجة المنفصلة وتحويل سلانتليت على حد سواء كما في مصفوفة D2، ولكن مختلطة من قبل المنتج تنسور. يتم تقييم معلمات الأداء مثل نسبة الضغط، نسبة ذروة الإشارة إلى الضوضاء، والجذر التربيعي لمتوسط الخطأ لكل من الصور الملونة والرمادية القياسية. وتبين نتيجة المحاكاة أن التقنيات توفر جودة الصور التي كانت طبيعية وهي مقبولة لكنها تحتاج إلى المزيد من العمل من قبل الباحثين.

الكلمات المفتاحية-تحويلات مختلطة، الموتر المشغل، المصفوفات متعامد، تحويل المويجات، تحويل سلانتليت، ضغط الصور.