Influence of Varying the Normal Load Position on Maximum Bending Strength of asymmetrical Helical Gear Tooth

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Abstract

The primary common causes of gear tooth failures are the bending stress on loaded gear tooth. wherein the gear tooth fillet is an area of maximum bending stress that occurs on the loaded and unloaded side. Theoretically, bending stress has been evaluated in standard helical involute gear which is designed for transformation of rotation between parallel axis based on classical method. Later, this equation has been modified in order to evaluate the effect asymmetric teeth on bending stress. This paper presents a numerical analysis by using finite element technique, accomplished basically by using SOLIDWORKS software 2016 package to investigate the influence of varying the normal load position on maximum bending stresses of asymmetrical helical gears tooth drive and comparing with those of the most common standard gear tooth. The aim of the numerical method is to interpolate an approximate solution to boundary value problem. The results of this work showed for an asymmetrical helical gear with teeth profiles having loaded side pressure angle of (14.5°) and unloaded side pressure angle of (35°) is better by about 14.278 % when the load acts at the tip of the tooth and about 29.582% when the load acts at the root of the tooth than a conventional helical gear from point of view of tooth bending strength. Also, there is a reduction 14.248% and 29.587% in the maximum tensile root fillet stresses when the total load position varies along the helical gear tooth profile from tip to root.

Keywords: Asymmetric helical gear tooth, Involute profile, Stress analysis, Bending stress, Solidworks Simulation.

1. Introduction

Helical gears are most widely utilized in transmission power and motion between parallel shafts, there be relatively smooth with silent operation due to large load carrying capacity in addition to higher operating speed[1]. Many numbers of researchers concentrated their investigations on the type of helical gear with asymmetric involute teeth profiles and presented a method to estimate the bending stress at the critical section based on the finite element method for this analysis. Himte et al,2012,[2] compared the bending stress of carrying capacity by an asymmetric gear and a symmetric gear where ANSYS package analysis has been used to calculate the bending stress of carrying capacity of gear tooth under a given loading condition. Sondur et al,2013[3] investigated the bending stress at the weakest section for asymmetric involute tooth gear theoretically by used the ISO/TC-60 method to calculate the bending stress with different pressure angle the results that the bending stress at the critical section was reduced by about 20% when drive side pressure angle was increased from 20° to 35°.Mishra,2013[4] compared the results obtained from both ANSYS and MATLAB Simulink to showed that the complex design problem for helical gear which that modelled on Pro/engineer package the result were close to the results that obtained from AGMA procedure to predicted the value of bending stress at any required face width. Gidado et al,2014,[5] presented the analytical investigation was based on the Lewis stress formula the results were compared with both AGMA and FEM procedures by used ANSYS package v.11, showed that there was a little variation with a higher difference in the percentage of 4.70% and the conclusion that ANSYS software could also be used to predict the values of bending stress with different the face width. Can be concluded fillet stresses as a function of tooth geometry and loading condition also finite element technique was an accurate method to determine the stress value numerically.

Helical gear has a helix angle that is produced from the axial twist of the teeth. It varies from tooth base to outside circle, the helix angle is denoted by (ψ) and is defined as the angle between the tangent of out circle of the tooth at the intersection of the pitch cylinder and the tooth profile[6]. As shown in **Fig.1.**

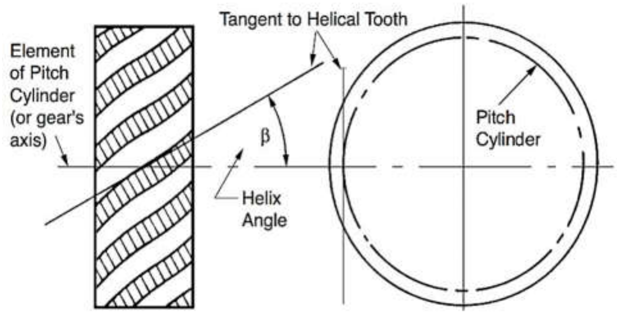
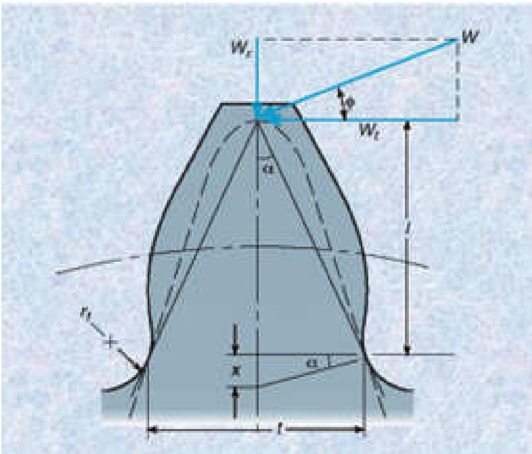


Fig.1 Definition of helix angle [6]

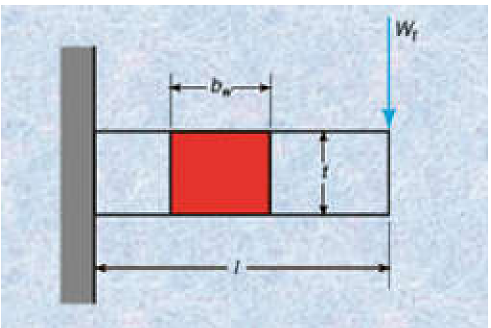
The teeth will have been oblique with the axis of rotation and as a result, have a considerable amount of overlap. The continuous contact for helical gear has a large load-carrying capacity and run will be high speed, more smoothly and quietly than straight spur gear of the same size. In the fact, the increase in total area of the average of the tooth during in contact, lead to that the load capacity for helical gears is greater than that of spur gears of the same size [7].

1.1. Formulation of The Loading Angle.

The hypothesizes which were used to estimate the theoretical solution of bending stress which that acting on the gear tooth based on the bending of a cantilever beam as shown in **Fig.2.**



a



b

Fig.2 Gear Tooth as Cantilever Beam [8]

The normal load ($\boldsymbol{W}$) acts with to the tooth surface at an angle and this angle is defined as loading angle and is denoted by ($\boldsymbol{\beta}$) as shown in **Fig(3).**

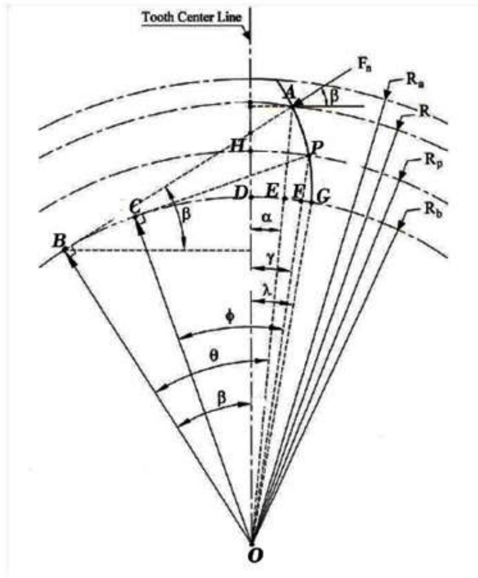


Fig.3 Determination of the loading angle β

In order to determine the angle ($\boldsymbol{\beta}$) between the force line of action and the horizontal line passing through point H. Referring to Fig(3).[9].

$$\beta = \theta - \alpha \qquad \ldots\ldots(1)$$

$$R_b = R_p\cos(\theta) \qquad \ldots\ldots(2)$$

$$R_b = R\cos(\theta) \qquad \ldots\ldots(3)$$

Hence

$$\theta = \cos^{-1}\left(\frac{R_p}{R}\cos\emptyset\right) \qquad \ldots\ldots(4)$$

Also from the properties of involute profile :

$$arc(HP) = \frac{P_c}{4} \qquad \ldots\ldots(5)$$

And $arc(HP) = R_p.\gamma$ ... (6)

But $P_c = \pi.m_0$ ... ... (7)

And $R_p = \dfrac{m_0.z}{2}$ ... ... (8)

R_p : represents the pitch radius.

From Eq.(5), Eq.(6), Eq.(7) and Eq.(8) it is clear that:

$$\gamma = \frac{\pi}{2.z} \quad \dots\dots\dots (9)$$

The angle (FOG), can be calculated as the follows:

$\overline{CP} = arc(CDEFG)$

Now, dividing by R_b to get ;

$$\frac{\overline{CP}}{R_b} = \frac{arc(CDEFG)}{R_b}$$

$$\tan(\emptyset) = \angle COG = \emptyset + \angle FOG$$

$$\angle FOG = \tan(\emptyset) - \emptyset \quad \dots\dots\dots (10)$$

Also, in the same way, to calculate $\angle EOG$,

$\overline{BA} = arc(BCDEFG)$

Now, dividing by R_b to get;

$$\frac{\overline{BA}}{R_b} = \frac{arc(BCDEFG)}{R_b}$$

$$\tan(\theta) = \angle BOG = \theta + \angle EOG$$

$$\angle EOG = \tan(\theta) - \theta \quad \dots\dots\dots (11)$$

Since;

$$\alpha = \gamma + \angle FOG - \angle EOG \dots (12)$$

From Eq.(10), Eq.(11) and Eq.(12) , it is clear that:

$$\alpha = \frac{\pi}{2.z} + (\tan(\emptyset) - \emptyset) - (\tan(\theta) - \theta) \dots (13)$$

Therefore, the loading angle β can be obtained as:

$$\beta = \tan\left[\cos^{-1}\left(\frac{R_p}{R} * \cos(\emptyset_l)\right)\right] - \frac{\pi}{2*z} - (\tan(\emptyset_l) - \emptyset_l) \dots\dots\dots\dots\dots (14)$$

Where the expression ($\emptyset_l$) is the pressure angle for the loaded side of the tooth.

In order to calculate the loading angle of any point of the force on the loaded side and at any radius according to above equation.

$$w_t = w * \beta \quad \dots\dots (15)$$

2. Designing of The Helical Gear.

The parametric equation has been used to create the involute curve in order to represent the tooth profile.The length of any portion of the curve can be calculated by using the following equation of involute curve:[10]

$$x = R_b(\cos(\vartheta) + \vartheta * \sin(\vartheta)) \dots (16)$$

$$y = R_b(\sin(\vartheta) - \vartheta * \cos(\vartheta)) \dots (17)$$

Where "ϑ" is the period at which the curve is drawn over the constructed circle. This equation has been modified to represent the asymmetric (loaded and unloaded sides) of tooth profile and will be shown in form as:

$$x(t) = R_{b_l}\left(cos\left(t + \left(\frac{\pi}{2}\right) - \lambda_l\right) + t * sin\left(t + \left(\frac{\pi}{2}\right) - \lambda_l\right)\right) \dots\dots (18)$$

$$y(t) = R_{b_l}\left(sin\left(t + \left(\frac{\pi}{2}\right) - \lambda_l\right) - t * cos\left(t + \left(\frac{\pi}{2}\right) - \lambda_l\right)\right) \dots\dots (19)$$

$$x(t) = R_{b_u}\left(cos\left(-t + \left(\frac{\pi}{2}\right) + \lambda_u\right) - t * sin\left(-t + \left(\frac{\pi}{2}\right) + \lambda_u\right)\right) \dots\dots (20)$$

$$y(t) = R_{b_u}\left(sin\left(-t + \left(\frac{\pi}{2}\right) + \lambda_u\right) + t * cos\left(-t + \left(\frac{\pi}{2}\right) + \lambda_u\right)\right) \dots\dots (21)$$

These equations are used to build 3D models for spur and helical gears by used the SOLIDWORKS 2016 software.

2.1. Cases studies.

In this paper, five different cases have been investigated to estimate the influence of varying the normal load position on maximum bending stress of symmetric and asymmetric helical gears teeth. **Table.1** contains the specifications which are used to create the models of gears created in SolidWorks 2016. Also, This includes the following;

The geometrical parameters that are constant of all cases are the module $m_o = 7$ mm, the tooth face width $b = 60$ mm and the number of teeth on the pinion and gear where $z_1 = z_2 = 14$ number of teeth.

Also, the speed ratio = 1. In this work five categories are investigated the first is the common case having symmetric teeth profiles with loaded and unloaded pressure angles $(20^{\circ} / 20^{\circ})$ and the other are non-standard cases having asymmetric teeth profiles with pressure angles 14.5° for unloaded side and deferent pressure angles variation from 20° to 35° for loaded side helical gears. In addition to, the helix angle has been selected to be $22.5^{\circ}$for helical gears cases. Because this value gives for models of helical gear having symmetric teeth that depend on the number of teeth (14number of teeth) and pressure angle (20°)[11]. For the purpose of right comparison between the models the helix angle $(\boldsymbol{\psi})$ is selected 22.5 for asymmetric teeth profiles.

Table.1 Case Studies

Case NO.	Case item	$\emptyset_l$	$\emptyset_u$	$h_a\,mm$	$h_d\,mm$	$r_f\,mm$
1	Standard Gear	20°	20°	m∘	1.166m∘	0.3m∘
2	Asymmetric Gear	14.5°	20°	m∘	1.25m∘	039m∘
3	Asymmetric Gear	14.5°	25°	m∘	1.25m∘	0.39m∘
4	Asymmetric Gear	14.5°	30°	m∘	1.25m∘	0.39m∘
5	Asymmetric Gear	14.5°	35°	m∘	1.25m∘	0.39m∘

A: Geometry

B: Loading and Boundary Condition

A normal load ($\boldsymbol{W}$) of *2400* N is applied with an angle loaded ($\boldsymbol{\beta}$) at four positions along involute tooth profile from tip to root point as followed:

Position (A): R1 = Ra = 60.6139 mm

Position(B): R2 = 56.284 mm

Position (C): R3 = Rp = 53.0372 mm

Position(D): R4= 51.345 mm

As for the boundary condition, the surface on the two sides of the three teeth of the gear rim is considered as a fixed constraint. As shown in **Fig(4).**

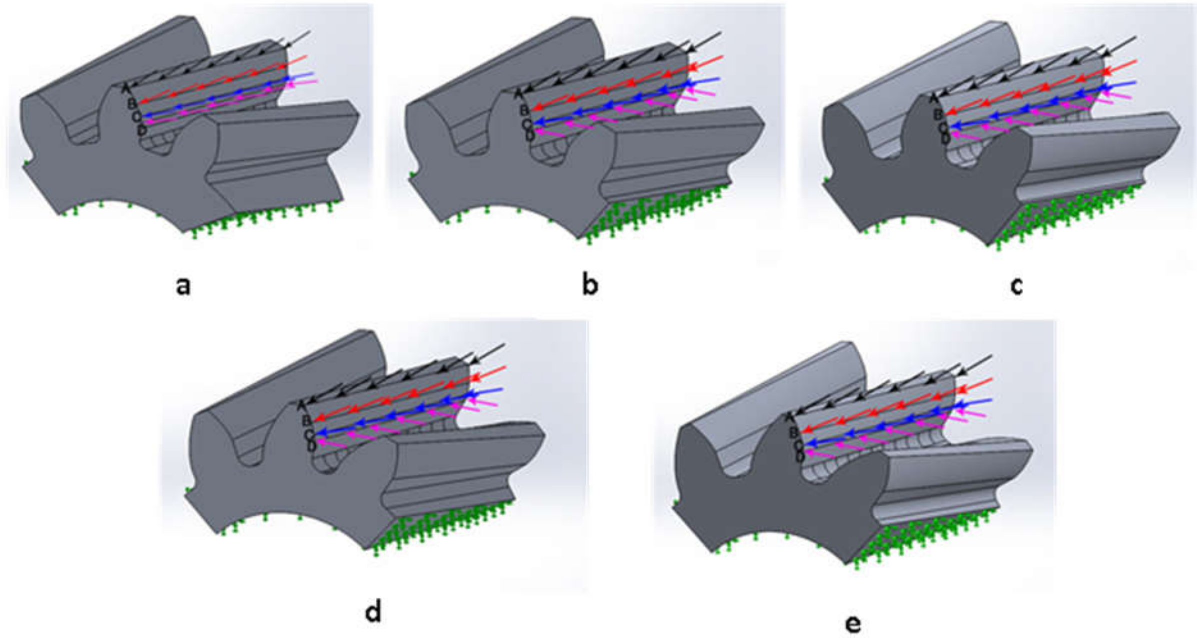


Fig.4 External normal force is applied with an angle at four position along involtue tooth profile for five models. (a) standard helical gear (20/20). (b) asymmetric tooth profile (20/14.5). (c) asymmetric tooth profile (25/14.5). (d) asymmetric tooth profile (30/14.5). (e) asymmetric tooth profile (35/14.5). with boundary condition.

C: Material

Aluminum alloy AA6061-T6 has been used in this investigation, with a modulus of elasticity, E = 68100 N/mm^2 and Poisson's ratio v = 0.33. Also, the material is assumed to be linear elastic.

3. Calculation Of The Loading Angle

In this work, Matlab code **v.**15.1 has been employed to determine loading angle of loaded side tooth side ($\boldsymbol{\beta}$) at any radial position ($\boldsymbol{R}$) for any pressure angle ($\emptyset$).**Table.2** represented the values of loading angles at four positions along involute tooth profile from tip to root.

And **Fig(4).** Represented the external normal force is applied with an angle at four positions along involute tooth profile for five models.

Table. 2 The value of loaded angles with radii position are calculated

Position	Loaded Angle For Standard Gear (deg)	Loaded Angle For Asymmetric Gear (deg)	Radii Position (mm)
Position (A)	32.378	29.194	60.6139
Position (B)	22.785	18.974	56.284
Position (C)	13.571	8.071	53.0372
Position (D)	6.928	-6.746	51.345

4. Numerical Analysis

In general, Bending strength which causes in helical teeth is very complex to calculate. because the line of action is placed at an angle on the tooth during the start and to end the engagement [12]. This problem may be solved by using FEM [13]. It can be concluded that the solving of the calculation of stresses on tooth flanks by the analytical methods is very complicated and possible only with many assumptions. The gear tooth will fail in bending when the total load that acting on the gear tooth greater than its strength. The Finite Element Method formulates the differential equations of the balance of an elastic body. By taking into consideration the boundary conditions, the number of unknown quantities in these equations becomes smaller (Gosselin and Cloutier, 1991). The tooth model has been adopted from generation computer program, **Fig.(5).** shows three teeth of generated helical gear.

Fig.5 Three Dimensional model with Mesh for standard Helical gear of helix angle 22.5º

4.1. Numerical Bending Example

The procedure of evaluation bending stress has been used 3D FEM analysis by SOLIDWORKS SIMULATION 2016 software. In this analysis has been used the static stress analysis with linear material properties. The three teeth model of helical gears have been prepared by SOLIDWORKS PROGRAM 2016 for analysis. The correct type of element type and the number has been carried out according to the convergence iteration. Where are Tetrahedral elements have been used and their element number is varying for each model based on helix angle. The absolute mesh size of an element has been specified from model mesh setting and generated a 3D mesh for this model. The maximum tensile stresses will occur at the root of the helical tooth. In this work, the mesh of area root of the tooth is refined to get more accurate results. The absolute mesh size of the element that equal (2.06393mm), the total number of elements that equal (89994)and new mesh size of surface refinement on the root area that equal (1.16103mm). which shows three teeth of helical gear in **Fig.(5).**

4.2. Finite Element Results

In **Fig.6** and **Fig.7** show the bending stress distribution inside the gear tooth for helical gears when acts at the tip and root point. In **Table.3**, illustrates the results of FEM represented the maximum tooth bending stresses due to the tangential force acts at four position on involute tooth profile. **Fig.8** describes the relationship between the maximum bending stresses with normal load position. It is clear that when asymmetric gear tooth profiles have been used there is a reduction in the bending stress by about 3.273%, 8.781% and 14.248 % for unloaded side pressure angle 25°, 30°and 35° respectively. And this reduction increased till it is about 13.335%,19.379%, and 29.587% when reaching the root position. Due to a great critical cross-section area in the asymmetric gear tooth profile. As for asymmetric teeth gear (20°/14.5°), the bending stress increases by about 18.661% and then reduces with the decreased normal load position due to undercut case.

Also, from **Fig.8**, it is clear that there is an improvement in the maximum bending stress at the tensile root fillet when the total load position varies along the gear tooth profile as shown in **Table.4** due to this varying in the load position will lead to varying the weakest-section position towards increasing the critical cross-section area of the gear tooth. **Fig.9** shows the relationship between the enhancement percentage relative to the standard case with normal load position. In **Table.5** shows the maximum enhancement percentage is 29.583% for asymmetric helical gear of unloaded sided pressure angle (35°) and loaded sided pressure angle (14.5°) with helix (45°) at root position and 14.278% at tip position.

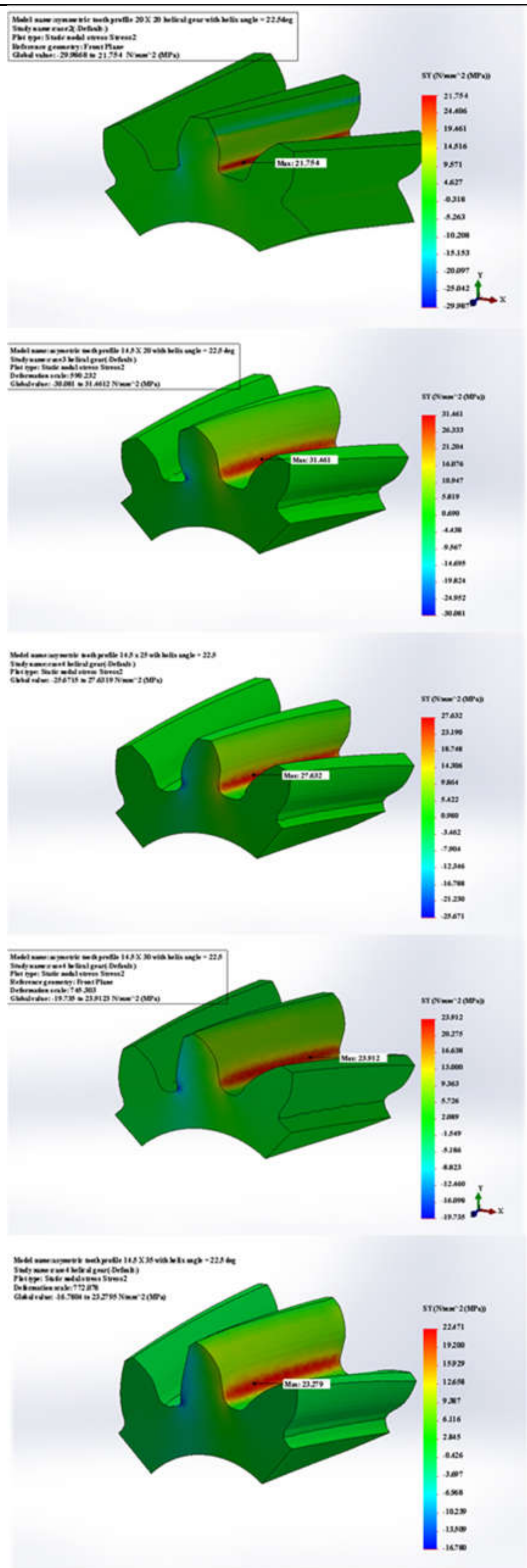


Fig.6 Three dimensional Bending Stress Distribution Inside Tooth Of Helical Gears When The Laod Acts At The Tip Point

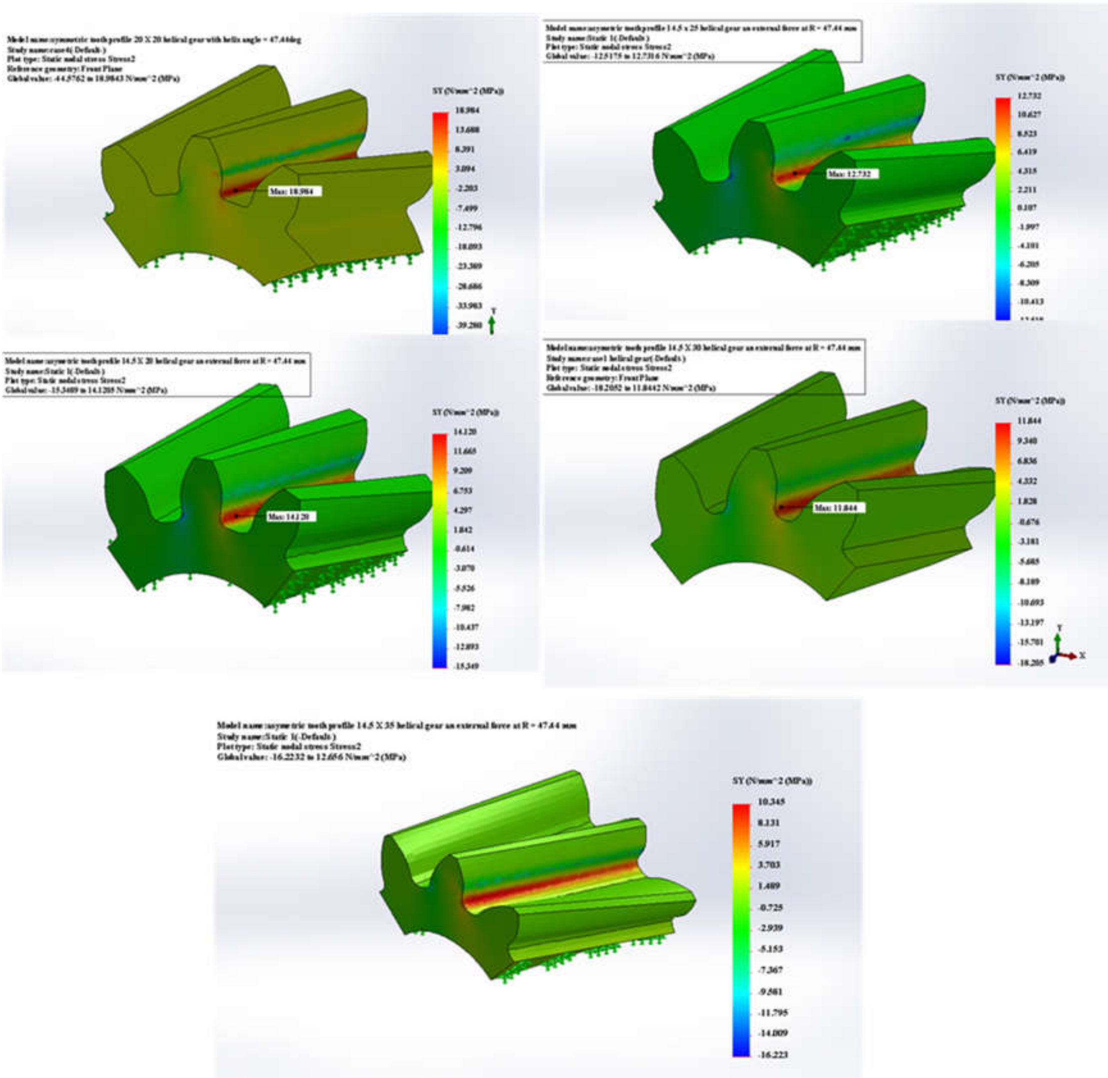


Fig.7 Three Dimensional bending stress distribution inside tooth of helical gears when the load acts at the root point

Table.3 FE results of maximum tooth bending stress of helical gears with helix angle = 22.5°

Position	$max\sigma_{bending}$(N/mm^2)				
		Asymmetric gear			
	20°/20°	20°/14.5°	25°/14.5°	30°/14.5°	35°/4.5°
A	26.214	31.106	25.356	23.912	22.471
B	21.754	22.859	20.470	18.180	17.367
C	17.353	17.216	15.649	14.256	13.734
D	14.691	14.120	12.732	11.844	10.345

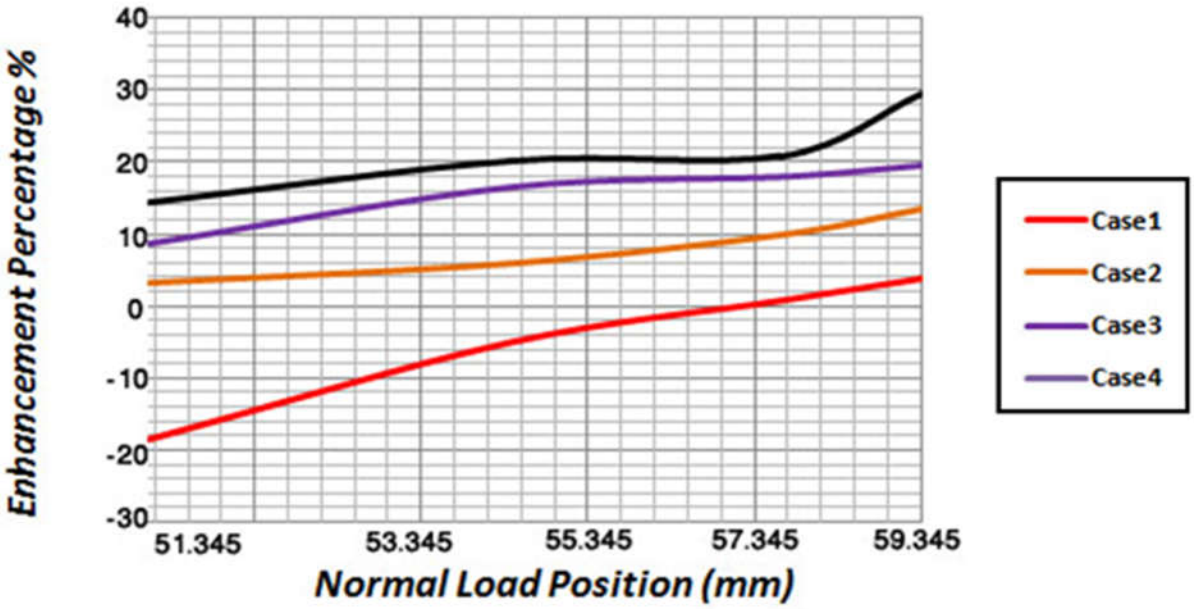


Fig.8 The maximum bending stress at the tensile root fillet when the normal load position varies along the gear tooth profile from tip to root.

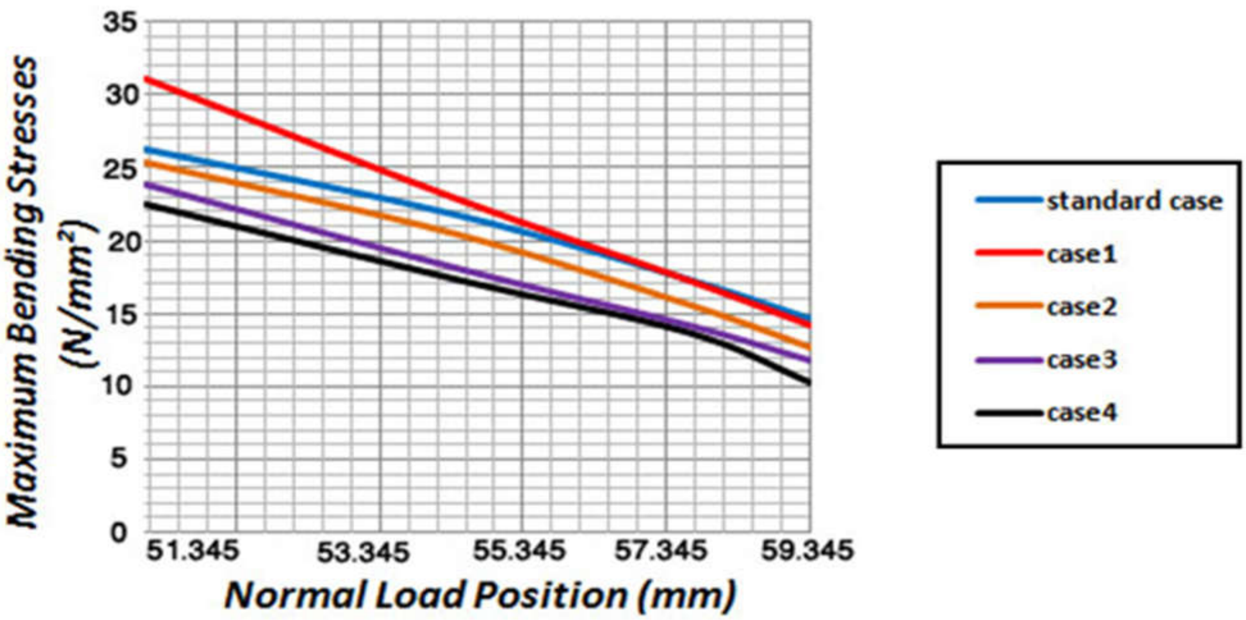


Fig.9 The relationship between the enhancement percentage relative to the standard case with normal load position. When the normal load position varies along the gear tooth profile from tip to root

Table.5 Results of enhancement percentage that compare with the standard gear, when the normal load position varies along the gear tooth profile from tip to root

Position	Enhancement Percentage %			
	Asymmetric Gear			
	$20^\circ/14.5^\circ$	$25^\circ/14.5^\circ$	$30^\circ/14.5^\circ$	$35^\circ/14.5^\circ$
A	-18.6618	3.27306	8.781567	14.27863
B	-5.07953	5.902363	16.42916	20.16641
C	0.789489	9.819628	17.84706	20.85518
D	3.886733	13.33469	19.37921	29.58274

5.Conclusions

The results obtained from this work can be as follows:

1- The maximum tooth bending stress was decreased and improved with the normal load position varies along the gear tooth from tip to root and by about 3.273%, 8.781% and 14.248 % for unloaded side pressure angle 25°, 30° and 35° respectively. And this reduction increased till it is about 13.335%,19.379%, and 29.587% when reaching the root position.

2- The modified helical gear which that having asymmetric tooth profiles with loaded side pressure angle (14.5°) and unloaded side pressure angle (35°) is better than a standard helical gear which that having symmetric tooth profile with standard pressure angle (20°) by about 14.278 % when the load acts at the tip of the tooth and about 29.582% when the load acts at the root of the tooth.

3- The influence of asymmetric tooth profile caused to the increased in the line of action because of using alow pressure angle of the loaded tooth side. Thereby, causing to the improvement in the bending stress distribution inside the gear tooth domain.

Notation

b: tooth face width (mm)

E: modulus of elasticity (N/mm)

W: normal applied force at the tip of tooth (N)

h_a , h_b: addendum and dedendum heights (mm)

m_0: module (mm)

m_t: transverse module (mm)

P_c: circular pitch (mm)

r_f: fillet radius (mm)

R_a: radius of addendum circle (mm)

R_b: radius of base circle (mm)

R_d: radius of dedendum circle (mm)

R_p: radius of pitch circle (mm)

Z: number of gear teeth

β: loading angle of the normal applied load (degree)

ψ: helix angle (degree)

v: Poisson's ratio

$\emptyset_l$, $\emptyset_u$: pressure angles for loaded and unloaded sides (degree)

β_l: loading angle of the normal applied load of loaded side tooth side

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تأثير اختلاف موضع الحمل العادي على مقاومة الانحناء القصوى لأسنان التروس الحلزونية غير المتماثل

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الخلاصة

الأسباب الشائعة الرئيسية لفشل اسنان التروس هو اجهاد الحناية على سن الترس المحمل. حيث ان منطقة جذر السن هي منطقة التي يكون فيها اجهاد الحناية اكبر ما يمكن و التي يتواجد على جانبي سن الترس المحملة و الغير محملة. نظريا، تم تقييم الانحناء الإجهاد في الترس الحلزوني القياسي الذي صمم لتحويل الدوران بين محور مواز اساس على الحسابات التقليدية. في وقت لاحق، تم تعديل هذه المعادلة لحساب الاجهادات الاسنان للتروس الغير متماثلة من أجل تقييم تأثير الأسنان غير المتماثلة على الانحناء الإجهاد. هذا البحث يقدم تحليلا للاجهادات التي تظهر على المسنن الحلزوني الغير متماثل عدديا باستخدام تقنيه العناصر المحددة باستخدام برنامج المحاكاة السولدورك 2016 لبحث تاثير تنوع موقع الاحمال و مقارنتها مع المسننات المتناظرة الاكثر شيوعا. النتيجة كانت ان المسننات المائلة مع اسنان غير متناظرة بزوايا ضغط (14.5°) للجانب المحمل و (35°) للجانب غير المحمل من السن هي افضل حوالي 14.278%عند تسليط الحمل عند نقطة قمة السن و حوالي 29.582% عند تسليط الحمل هند نقطة جدر السن من المسننات المائلة القياسية . كما ان هناك انخفاض في اجهاد الشد الحاصل في منطقة جذر السن تقدر حوالي 14.248% و 29.587% على التوالي عندما يتنوع موقع القوة الكلية على طول سن الترس المائل من القمة الى الجذر.

الكلمات المفتاحية: اسنان الترس الحلزوني الغير متماثلة، تحليل الاجهادات، تحليل اجهاد الحناية، محاكاة السولدورك 2016