https://doi.org/10.33261/jaaru.2019.26.1.006 *Association of Arab Universities Journal of Engineering Sciences (2019) 26(1): 44–48*

Association of Arab Universities Journal of Engineering Sciences

مجلة اتحاد الجامعات العربية للدراسات والبحوث الهندسية

A New Mathematical Analysis of Two–Plane Balancing for Long Rotors without Phase Data

Mohammed Tariq Sulaiman Al Abbood

Department of Mechanical Engineering, University of Mosul, Mosul, Iraq, mohammed.alabbood.altaee@uomosul.edu.iq

Published online: 31 March 2019

Abstract— Two–plane balancing procedure is usually adopted to balance long rotors using both vibration amplitude and phase data. This paper presents a new mathematical analysis of two–plane balancing method for long rotors using vibration amplitude data only. This method requires eight test runs with two balancing planes nearby the bearings. The tests are performed by connecting a known trial mass to eight different positions individually; four at each plane, where each position being advanced (90^{o}) from its previous. In this study, a comparison has been performed between the new mathematical analysis and another traditional analysis presented in a previous study. Firstly, two computer programs based on both analyses have been written using C^{++} Language in order to compute the magnitudes and locations of the required balancing masses. Secondly, the comparison has been made using balancing simulator for rigid rotors where different sizes of rotors at different rotation speeds have been tested. This study showed that the two–plane balancing method based on the new analysis was always capable of performing a very high grades of balance while traditional analysis showed an observed restriction in achieving a good quality of balance for the rotors been tested.

Keywords—Balancing without phase data, balancing of long rotors, two–plane balancing using vibration amplitude data.

1. Introduction

Vibration and accompanying problems such as noise and fatigue are considered as the main factors that decrease the performance of rotating machines, so the efforts to overcome such vibration are becoming more essential. Vibration in rotating machines is a result of different mechanical drawbacks including mass unbalance, coupling misalignment, components looseness and other many reasons. However, mass unbalance is probably the most common source of extreme vibration in rotating machines.

Over the last eighty years, different balancing methods have been presented to minimize vibration of rotors caused by unbalance. The initial research that dealt with rotor vibration due to unbalance was traced back to the 1930s. Thearle [1] formulated a two–plane technique using the influence coefficients method. Goodman [2] presented the least–squares algorithm, an extension of the influence coefficients method, for balancing of flexible rotors using amplitude and phase data collected from multiple speeds and measuring positions. Kang et al. [3] presented a modified influence coeffcients method to balance asymmetrical rotors such as crankshafts using soft–pedestal machines. The accuracy and validity of the modified method were checked both theoretically using compueter simulation and practically through several balancing experiments on real crankshafts. The modified approach yielded a better quality of balancing than the conventional balancing method did. Sinha et al. [4] estimated a method to balance a rotor–bearing–foundation system using amplitudes and phases data measured at the bearing pedestals. The proposed method was applied to an experimental test rig where it showed excellent results. Al–Taee [5] presented both graphical and mathematical analyses for two–plane balancing of rigid rotors and other two mathematical analyses for three–plane balancing of flexible rotors. Firstly, the computer programs that related to the mathematical analyses were written then their validity on an experimental test rig of a long rotor was successfully checked.

On the other hand, in some applications, balancing must be performed without using phase data, this could be simply because of absence of phase measuring devices or because the machine rotating parts needed to be balanced are completely bounded or not easy to be reached. In such cases, a high quality of balance can be obtained using vibration meter only. Wilcox [6] presented a graphical solution for single–plane balancing using amplitude data taken from five runs, one due to original unbalance and four due to trial mass. The method of Wilcox is also known as the four runs method.

In addition, analytical studies of balancing using amplitude only has had several extensions. Nisbett [7] presented a mathematical analysis of the two–plane balancing of rigid rotors using amplitude data only. It stated that the technique can be used in the field efficiently although it takes eight trial mass runs. Al–Taee [8] significantly simplified the graphical solution of the four runs method by presenting a new mathematical analysis. The validity of the new analysis was successfully experimented on a disk shaped rotor. Ali et al. [9] presented a mathematical analysis of the graphical single–plane balancing method, also known as the three runs method, using amplitude data taken from runs of original unbalance and three trial mass. The validity of their analysis was investigated on an experimental test rig of a narrow rotor where excellent results were obtained.

Al–Abbood [10] presented a new single–plane balancing method without phase data using only two runs of trial mass. The method's validity was checked on a crankshaft of a domestic electrical generator. The proposed method was very active, practical and saving a lot of cost, time and efforts since just two runs of trial mass are required. Han et al. [11] presented a virtual prototyping technology to simulate mass unbalance and examine dynamic balancing of rigid rotors. The contribution of thier work was to provide a new way to verify balancing method and analyze balancing error without a real test. The results validated the correctness and feasibility of the proposed method. Sampaio and Silva [12] presented two virtual experiments that can be used to study field balancing of rigid rotors and to train its implementation. The simulator can provide surprisingly accurate vibrations data for static and dynamic unbalance. The software has the ability to generate reasonable vibration data that makes easier to understand unbalance symptoms and balancing methods without the need for any physical models.

This paper presents a new mathematical analysis for the two–plane balancing procedure of long rotors using vibration amplitude data only. The remainder of this paper is organized as follows. Section 2 presents the new mathematical analysis. The simulation work is given in section 3 where the author compared his proposed analysis with a traditional analysis of a previous study. Section 4 presents the results and discussion. Finally, the conclusions are presented in the last section.

2. Mathematical Analysis

A sketch of one of the long rotors that will be balanced using two–plane algorithm is shown in Fig. 1.

In the two–plane balancing method, each plane requires four individual runs of trial mass at four angles; 0^o, 90^o, 180^o and 270^o where the reference or zero degree is randomly chosen.

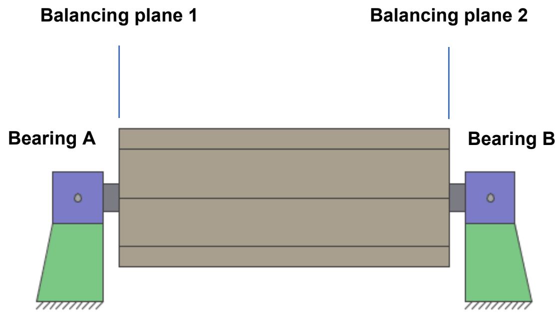

Figure 1: Sketch of a long rotor

Using these measurements, the four vector diagrams related to both planes can be constructed. Fig. 2 shows one of the four diagrams, which is related to bearing A due to trial mass runs at plane 1.

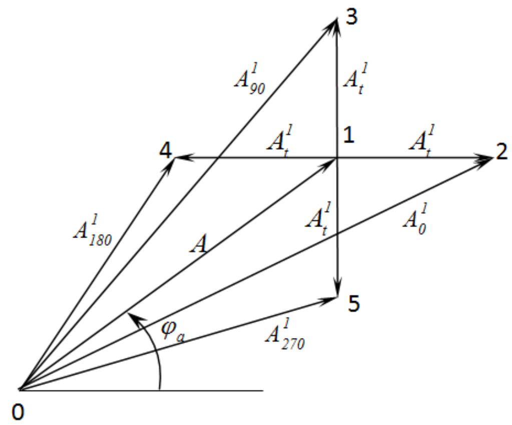

Figure 2: Vector diagram of bearing A due to trial mass runs at plane 1

From triangles 012, 013, 014 and 015 in the vector diagram shown in Fig. 2 and by using cosine law, it can be found out that [7, 8]

$$(A_0^1)^2 = (A)^2 + (A_t^1)^2 + 2 \times A \times A_t^1 \times cos\,\phi_a \quad (1)$$

$$(A_{90}^1)^2 = (A)^2 + (A_t^1)^2 + 2 \times A \times A_t^1 \times sin\,\phi_a \quad (2)$$

$$(A_{180}^1)^2 = (A)^2 + (A_t^1)^2 - 2 \times A \times A_t^1 \times cos\,\phi_a \quad (3)$$

$$(A_{270}^1)^2 = (A)^2 + (A_t^1)^2 - 2 \times A \times A_t^1 \times sin\,\phi_a \quad (4)$$

where A_0^1, A_{90}^1, A_{180}^1 and A_{270}^1 are the vibration amplitudes at bearing A due to trial mass mounted on plane 1 at angle 0^o, 90^o, 180^o and 270^o respectively, A is the vibration amplitude at bearing A due to original unbalance of the rotor, A_t^1 is the vibration amplitude at bearing A due to trial mass effect only, mounted at any angle on plane 1, ϕ_a is the phase angle of original unbalance at bearing A.

Now, by subtracting Eq. (3) from Eq. (1) and Eq. (4) from Eq. (2) then dividing the obtained equations by each other, the value of ϕ_a can easily be found as:

$$\phi_a = tan^{-1}\left[\frac{(A_{90}^1)^2 - (A_{270}^1)^2}{(A_0^1)^2 - (A_{180}^1)^2}\right] \quad (5)$$

The phase angle ϕ_a can also be written as:

$$\phi_a = tan^{-1}\left[\frac{(A^2_{90})^2-(A^2_{270})^2}{(A^2_0)^2-(A^2_{180})^2}\right] \quad (6)$$

where A^2_0, A^2_{90}, A^2_{180} and A^2_{270} are the vibration amplitudes at bearing A due to trial mass mounted on plane 2 at angle 0^o, 90^o, 180^o and 270^o respectively.

The value of A^1_t can be written as a function of A^1_0 and A^1_{180} or as a function of A^1_{90} and A^1_{270} such that:

$$A^1_t = \left[\frac{(A^1_0)^2-(A^1_{180})^2}{4*A*cos\,\phi_a}\right] = \left[\frac{(A^1_{90})^2-(A^1_{270})^2}{4*A*sin\,\phi_a}\right] \quad (7)$$

Similarly, A^2_t can be derived from its related vector diagram as a function of A^2_0 and A^2_{180} or as a function of A^2_{90} and A^2_{270} such that:

$$A^2_t = \left[\frac{(A^2_0)^2-(A^2_{180})^2}{4*A*cos\,\phi_a}\right] = \left[\frac{(A^2_{90})^2-(A^2_{270})^2}{4*A*sin\,\phi_a}\right] \quad (8)$$

where A^2_t is the vibration amplitude at bearing A due to trial mass effect only, mounted at any angle on plane 2.

In the same way, the three main values which belong to bearing B, namely ϕ_b, B^1_t and B^2_t can be derived as:

$$\phi_b = tan^{-1}\left[\frac{(B^1_{90})^2-(B^1_{270})^2}{(B^1_0)^2-(B^1_{180})^2}\right] \quad (9)$$

or

$$\phi_b = tan^{-1}\left[\frac{(B^2_{90})^2-(B^2_{270})^2}{(B^2_0)^2-(B^2_{180})^2}\right] \quad (10)$$

where ϕ_b is the phase angle of original unbalance at bearing B, B^1_0, B^1_{90}, B^1_{180} and B^1_{270} are the vibration amplitudes at bearing B due to trial mass mounted on plane 1 at angle 0^o, 90^o, 180^o and 270^o respectively, B^2_0, B^2_{90}, B^2_{180} and B^2_{270} are the vibration amplitudes at bearing B due to trial mass mounted on plane 2 at angle 0^o, 90^o, 180^o and 270^o respectively.

The value of B^1_t can be written as a function of B^1_0 and B^1_{180} or as a function of B^1_{90} and B^1_{270} such that:

$$B^1_t = \left[\frac{(B^1_0)^2-(B^1_{180})^2}{4*B*cos\,\phi_b}\right] = \left[\frac{(B^1_{90})^2-(B^1_{270})^2}{4*B*sin\,\phi_b}\right] \quad (11)$$

where B^1_t is the vibration amplitude at bearing B due to trial mass effect only, mounted at any angle on plane 1.

Similarly, the value of B^2_t can be written as a function of B^2_0 and B^2_{180} or as a function of B^2_{90} and B^2_{270} such that:

$$B^2_t = \left[\frac{(B^2_0)^2-(B^2_{180})^2}{4*B*cos\,\phi_b}\right] = \left[\frac{(B^2_{90})^2-(B^2_{270})^2}{4*B*sin\,\phi_b}\right] \quad (12)$$

where B^2_t is the vibration amplitude at bearing B due to trial mass effect only, mounted at any angle on plane 2.

The aim of calculating the values ϕ_a, ϕ_b, A^1_t, A^2_t, B^1_t and B^2_t is ultimately to determine the balancing masses M_a and M_b and their locations on planes 1 and 2 respectively which will eliminate the original unbalances at both bearings.

Reference [7] presented a different concept regarding values ϕ_a, ϕ_b, A^1_t, A^2_t, B^1_t and B^2_t. In addition, values A^1_t, A^2_t, B^1_t and B^2_t have been expressed by squared roots which leads, in many cases, to imaginary values and ends up with infinite solution. The main advantage of the new mathematical analysis is to overcome this problem and to get the exact solution for mass imbalance under any condition as the values A^1_t, A^2_t, B^1_t and B^2_t in the new analysis are free of any roots as shown in Eqs. (7), (8), (11) and (12) respectively. In this paper, a simulation comparison between both analyses has been made to know the effect of this difference between these two sets of values on balancing process where two C^{++} computer programs for both analyses have been developed.

The purpose of the trial mass runs is to find the correlations between trial and balancing masses. These correlations can be simply written in x and y components as [7]

$$M_{ax} = C_{ax} \times M_{trial} \quad (13)$$

$$M_{ay} = C_{ay} \times M_{trial} \quad (14)$$

$$M_{bx} = C_{bx} \times M_{trial} \quad (15)$$

$$M_{by} = C_{by} \times M_{trial} \quad (16)$$

where M_{trial} is the trial mass, C_{ax}, C_{ay}, C_{bx} and C_{by} are defined as the correction factors to be applied to the trial masses to get the four components masses required for balancing, M_{ax}, M_{ay}, M_{bx} and M_{by}. The four correction factors C_{ax}, C_{ay}, C_{bx} and C_{by} can be written as [7]

$$C_{ax} = \frac{B*A^2_t*cos\,\phi_b - A*B^2_t*cos\,\phi_a}{A^1_t*B^2_t - A^2_t*B^1_t} \quad (17)$$

$$C_{ay} = \frac{B*A^2_t*sin\,\phi_b - A*B^2_t*sin\,\phi_a}{A^1_t*B^2_t - A^2_t*B^1_t} \quad (18)$$

$$C_{bx} = \frac{-A*cos\,\phi_a - C_{ax}*A^1_t}{A^2_t} \quad (19)$$

$$C_{by} = \frac{-A*sin\,\phi_a - C_{ay}*A^1_t}{A^2_t} \quad (20)$$

The four component masses required for balancing can be determined now from Eqs. (13), (14), (15) and (16) respectively. In order to place these masses at their correct locations, x component should be placed at angle 0^o if positive and placed at angle 180^o if negative whereas y component should be placed at angle 90^o if positive and at angle 270^o if negative. However, in this work, the computer programs for both analyses have been developed so that these four component masses can be expressed into their resultant balancing masses M_a and M_b and their positions on the rotor.

3. Simulation Work

The aim of simulation work is to carry out the comparison between the new mathematical analysis presented in this paper and the traditional mathematical analysis presented in [7]. The simulation has been performed using balancing simulator for rigid rotors prepared by CBM Apps [13] as shown in Fig 3.

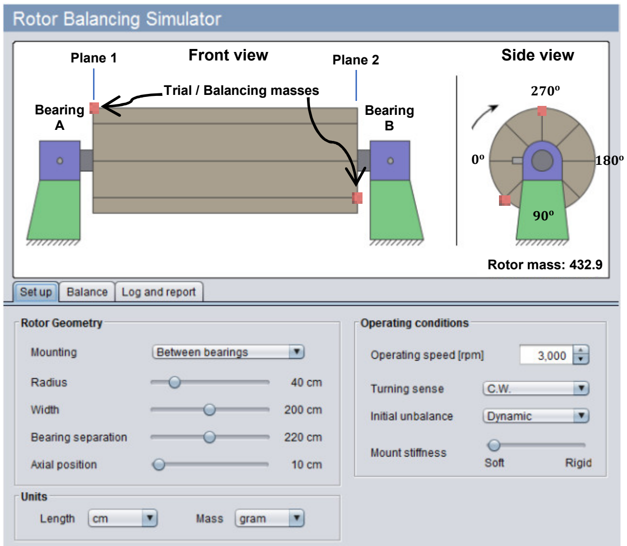

Figure 3: Panel of rotor balancing simulator

The panel of the simulator shows both front and side views of the rotor, rotor mass, rotor geometry including mounting type (between bearings or overhang), radius, width (length) and axial position from Bearing A. The panel also contains the operating conditions of the rotor which include speed, turning sense (clockwise or counterclockwise direction), type of initial unbalance (static or dynamic) and mount stiffness (soft or rigid). In addition, the units of length and mass of rotor could be chosen in SI or British system. Balancing process is performed through Balance button where trial and balancing masses can be mounted on planes of balancing. The final report of balancing process which includes vibration values for both bearings before and after balancing can be obtained through Log and Report button. Finally, it should be mentioned that this simulation is based on ISO 10816 standard.

4. Results and Discussion

Table 1 shows the measured vibration values at bearings before and after balancing for different rotor sizes at different speeds using the CBM Apps simulation software. As the simulator is based on ISO 10816 standard, it is designed to always mimic an imbalance state of rotors when it runs so no masses were needed to be added to create an initial imbalance conditions which gives the balancing process more reliability. Obviously, Table 1 confirms that the new mathematical analysis, gray shaded cells, showed that vibration values due to original unbalances at both bearings have dropped dramatically after balancing for the whole nine tested rotors where an improvement range of $80.4\% - 97.3\%$ has been obtained. On the other hand, the traditional analysis, white cells in Table 1, showed an observed restriction in achieving a good quality of balance for the rotors been tested where just one rotor out of the nine has been successfully balanced, five rotors cannot be balanced and end with no solution due to imaginary roots, and three rotors became worse through increasing of vibration after balancing.

Table 1: Measured vibration values at both bearings before and after balancing for different rotor sizes at different test speeds using balancing simulator

RPM	Vibration value at bearing A mm/s			Vibration value at bearing B mm/s		
	BB	AB	PI	BB	AB	PI
First rotor size: L = 2, D = 0.4, S = 2.2, M = 138						
1500	9.61	0.47	95.11	8.95	0.70	92.18
		NS	NS		NS	NS
3000	11.22	0.47	95.81	9.78	0.74	92.43
		NS	NS		NS	NS
5000	11.24	1.63	85.5	12.01	1.38	88.51
		0.19	98.31		0.71	94.09
Second rotor size: L = 2.4, D = 0.4, S = 2.6, M = 164						
1500	11.61	1.05	90.95	11.88	0.32	97.31
		1.43	87.68		22.8	NI
3000	12.53	0.86	93.13	12.60	2.42	80.79
		24.37	NI		24.28	NI
5000	11.95	0.40	96.65	9.34	1.83	80.40
		24.09	NI		19.53	NI
Third rotor size: L = 3, D = 0.6, S = 3.2, M = 382						
1500	10.29	1.04	89.89	10.23	1.10	89.24
		NS	NS		NS	NS
3000	11.17	1.26	88.72	11.40	1.00	91.23
		NS	NS		NS	NS
5000	12.62	1.54	87.80	11.35	2.00	82.38
		NS	NS		NS	NS

5. Conclusions

This study presented a new mathematical analysis of two–plane balancing method for long rotors based on amplitudes data only. This analysis has been examined and virtually compared with a traditional analysis presented in a previous work using balancing simulator for rigid rotors. The comparison showed that the new analysis presented in this study was capable of achieving a high quality of balance for the rotors been tested from the first balancing attempt. On the other hand, the traditional analysis showed an observed restriction represented in many cases by imaginary roots which ends up with no solution or even worse through increasing of vibration value. The main advantage of the new mathematical analysis is its ability to overcome this problem through presenting a new expression without roots. It can be concluded that although the traditional analysis showed its limitations to balance long rotors, it is still able to balance relatively short or disk-shaped rotors.

References

[1] E. L. Thearle, "Dynamic balancing of rotating machinery in the field", ASME Journal of Applied Mechanics, vol. 56, no. 19, p. 745–753, 1934.
[2] T. P. Goodman, "A least–squares method for computing balance corrections," Trans. ASME Journal of Engineering for Industry, vol. 86, no. 3, p. 273–279, 1964.

[3] Y. Kang, Y. P. Chang, M. H. Tseng, P. H. Tang, and Y. F. Chang, "A modified approach based on influence coefficient method for balancing crankshafts", Journal of Sound and Vibration, vol. 234, no. 2, p. 277–296, 2000.
[4] J. K. Sinha, A. W. Lees, and M. I. Friswell, "Estimating unbalance and misalignment of a flexible rotating machine from a single run–down," Journal of Sound and Vibration, vol. 272, no. 3-5, p. 967–989, 2004.
[5] M. T. Al–Taee, "Dynamic balancing of rotating shafts by vibration measurements," M.Sc. thesis, Dept. of Mech. Eng., Baghdad Univ., Baghdad, Iraq, 2004.
[6] J. B. Wilcox, "Dynamic balancing of rotating machinery," Nottingham Regional College of Technology, Sir Isaac Pitman and Sons Ltd., London, England, 1967.
[7] K. Nisbett, "Dynamic balancing of rotating machinery experiment," Missouri University of Science and Technology, Laboratory Experiment, p. 53–61, 1996. http://web.mst.edu/~stutts/ME242/LABMANUAL/DynamicBalancingExp.pdf
[8] M. T. Al–Taee, "Theoretical and experimental method of rotating discs balancing," Al–Rafidain Engineering Journal, vol. 16, no. 1, p. 89–101, 2008.
[9] S. J. Ali, M. T. Al–Taee, and, G. I. Al–Sarraj, "Analytical and experimental study of three–test run balancing method," Al–Rafidain Engineering Journal, vol. 17, no. 2, p. 77–85, 2009.
[10] M. T. Al–Abbood, "A novel experimental study of single–plane balancing method of crankshaft without phase angles data," Al–Rafidain Engineering Journal, vol. 21, no. 3, p. 66–77, 2013.
[11] T. Han, J. Bai, and, Z. Yin, "Dynamic balancing simulation based on virtual prototyping technology," 8th International Conference on Reliability, Maintainability and Safety, Chengdu, China, 2009.
[12] R. Sampaio, and T. Silva, "Unbalance and field balancing virtual labs," 11th IEEE International Conference on Remote Engineering and Virtual Instrumentation, p. 75–76, Porto, Portugal, 2014.
[13] CBM Apps, "Rotor balancing simulator," http://www.cbmapps.com/apps/35

Nomenclature

L	Length of rotor (m)
D	Diameter of rotor (m)
S	Span between bearings (m)
M	Mass of rotor (kg)

Abbreviations

RPM	Revolution per minute
BB	Before balancing
AB	After balancing
PI	Percentage improvement
NS	No solution (imaginary root)
NI	No improvement (vibration increased after balancing)

تحليل رياضيٌّ جديد للموازنة ذات المستويَيْن للدّوّارات الطويلة بدون بيانات الطّور

محمد طارق سليمان العبود

قسم الهندسة الميكانيكية، جامعة الموصل، الموصل، العراق، mohammed.alabbood.altaee@uomosul.edu.iq

نشر في: 31 آذار 2019

الخلاصة – إنّ الموازنة ذات المستويين عادةً ما يتم اعتمادها لموازنة الدوّارات الطويلة وذلك باستخدام كلٍّ من بيانات سعة الإهتزاز والطور. يقدم هذا البحث تحليلاً رياضياً جديداً لطريقة الموازنة ذات المستويين للدوّارات الطويلة باستخدام بيانات سعة الإهتزاز فقط. تتطلب هذه الطريقة ثماني اختبارات تجريبيّة بمستويين اثنين للموازنة قرب المحامل. تتم هذه الإختبارات عن طريق تثبيت كتلة تجريبيّة معلومة المقدار في ثمانية مواضع مختلفة، كلاً على حده، أربعة في كل مستوي، حيث يتقدم كل موضع بزاوية مقدارها 90 درجة عن سابقه. في هذه الدراسة، تم إجراء مقارنة بين التحليل الرياضيّ الجديد وتحليل تقليديّ تم تقديمه في دراسة سابقة. أولاً، تم كتابة برنامجين حاسوبيين لكلا التحليلين باستخدام لغة (C^{++}) من أجل حساب مقادير ومواقع الكتل اللازمة للموازنة. ثانياً، تمت المقارنة بين التحليلين من خلال استخدام تطبيق محاكاة لموازنة الدوّارات الطويلة حيث تم اختبار دوّارات ذات أحجام مختلفة وعند سرع مختلفة. لقد أظهرت هذه الدراسة أنّ طريقة الموازنة المبنيّة على التحليل الجديد كانت دائماً قادرةً على تحقيق درجاتٍ عاليةٍ جداً من الإتزان بينما أظهر التحليل التقليديّ قصوراً واضحاً في تحقيق جودة موازنة جيّدة للدوّارات التي تم اختبارها.

الكلمات الرئيسية – الموازنة بدون بيانات الطور، موازنة الدوّارات الطويلة، الموازنة ذات المستويين باستخدام بيانات سعة الإهتزاز.