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Behavior of Plain Concrete Beam Analyzed Using Extended Finite Element Method

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Abstract— In this study, plain concrete simply supported beams subjected to two points loading were analyzed for the flexure. The numerical model of the beam was constructed in the meso-scale representation of concrete as a two phasic material (aggregate, and mortar). The fracture process of the concrete beams under loading was investigated in the laboratory as well as by the numerical models. The Extended Finite Element Method (XFEM) was employed for the treatment of the discontinuities that appeared during the fracture process in concrete. Finite element method with the feature standard/explicitlywas utilized for the numerical analysis. Aggregate particles were assumedof elliptic shape. Other properties such as grading and sizes of the aggregate particles were taken from standard laboratory tests that conducted on aggregate samples.Two different concrete beamswere experimentally and numerically investigated. The difference between beams was concentrated in the maximum size of aggregate particles. The comparison between experimental and numerical results showed that themeso-scale model gives a good interface for the representing the concrete models in numerical approach. It was concluded that the XFEM is a powerful technique to use for the analysis of the fracture process and crack propagation in concrete.

Keywords— Extended finite element method, Flexural strength of concrete, Fracture mechanics, Meso-scale modeling, Two point loading.

1. Introduction

The behavior of non-homogenous materials like concrete is a result of the behavior of its components. Concrete is a three phasic material comprises of aggregate, mortar, and the interface between them. Mortar usually contains air voids, that conducted from the pouring process of concrete. The mechanical behavior of a material is a result of the small particles gradually up to the larger particles, i.e. the material will be affected by external loading from the atomic, micro, meso, to macro scale, which is can be seen by the naked eye. Most of concrete experiments and studies on all its fields based on the macro scale. To comprehend a material attitude due to external effects, its behavior should be studied in finer scales.

In this paper, plain concrete beam subjected to two point loads will be analyzed in a meso-scale numerical model, which will be partitioned into two phases aggregate and mortar. The results of this model will be compared with the experimental results of the same plain concrete beam with approximately the same composition. The interface between the aggregate and cement mortar in the numerical model was neglected and the two materials is assumed fully consistent.

A number of meso-scale model researches were conducted, e.g. [6], [15], and [10]. In general, meso-scale modeling can be categorized into two types: the continuum models and the lattice models. In the continuum model, concrete is modeled as a continuum composite material consist of aggregate, mortar, and interface zone between the two materials that ranged between20 – 100 μm[8].On the other hand, in lattice model, concrete is modeled as a discrete system consist of a lattice element [13]. Broadly, the lattice modeling method requires a huge numerical effort to obtain concrete meso structure needed for the analysis.

In this study, concrete was modeled as a continuum meso-scale model, consist of aggregate and cement mortar. The meso-scale modeling has two approaches; image based modeling and the parameterization modeling. The image based method rely on a set of two dimensional pictures that are assembled together to have a three dimensional model, then the numerical model will be conducted based on this three dimensional model. In general, the image based method is an accurate and precise method for the modeling of concrete than the conventional numerical models, but it is more expensive and time consuming method[4]. The parameterization approach can be classified into direct and indirect methods. In indirect method, concrete is randomly modeled with a suitable finite element mesh, or by using the lattice modeling for aggregate and mortar [16], and [18]. In the direct method, aggregate particles can be modeled with various physical properties such as shape, size, orientation, and these particles are floating in mortar, which is more convenient for the meso-scale modeling process. In this paper, the parametrization approach was considered for the modeling of concrete.

When concreteis applied to external loads, cracks will be gradually propagate. For the numerical analysis of cracked concrete, many techniques were conducted. The smeared crack method is one these technique introduced byRashid [14],in this method the stress in the individually finite elements is limited to the maximum tensile strength of the material, when this stress reach its limits the stress will be decreased. In general, this method relates the tangent of the strain-softening curve to the finite element size. Another traditional numerical technique is the re-meshing method [17]. In this method, the model is the re-meshed near the crack tip for every crack propagation step, obviously this technique is a time consuming technique.

In the last two decades, new methodswere developed for analyzing of crack propagation in concrete. These methods depend on the concept of enrichment functions. One of these methods is the (XFEM) that developed by Belytschko & Black in 1999[3] which utilizes the Partition of Unity (PU) technique which was developed byMelenk & Babuska, 1996[11] and involves the Level Set Method (LSM) for the numerical representation of crack propagation.

In this paper, a plain concrete simply supported beam is analyzed using a meso-scale model as a bi-phasic material consisting of aggregate and mortar, and the XFEM was utilized for the analyzing of discontinuity occurred during the crack propagation. ABAQUS program was used for the numerical analyzing. Laboratory specimens were prepared and tested for the comparison of results and behavior of the concrete beam. The beams in both numerical and experimental investigations were subjected to flexural stresses by applying two-point loading.

2. Basis of the Extended Finite Element Method

In structure engineering, concrete is the most common material used for the construction of various and multi purposes structures. During the life time of the structure, concrete is subjected to various types of loading. These load applications result in various categories of stresses on the overall structure and on the concrete individually. When tensile stresses reach the limit of concrete strength in tension, concrete will exhibit a multi fracture behavior that leads to the cracking of concrete. In numerical analysis, crack simulation is a complex issue, due to the discontinuity problems that appear in the numerical solution. One of the powerful method to solve this matter is the XFEM [7], which was utilized for the numerical analysis in this paper.

The XFEM is a numerical technique that apply the concept of the PU through the enrichment functions,at a region of the domain where discontinuity occurred through increasing the nodes of that region with the enrichment functions. The finite element approximation after enrichment function was involved is in Equation (1) and (2) present below:

$$u(x) = \sum_{i=1}^{N} N_i(x)\bar{u}_i + enrichment\ terms \quad (1)$$

$$u(x) = \sum_{i=1}^{N} N_i(x)\bar{u}_i + \sum_{i=1}^{N} \bar{N}_i(x)\left(\sum_{j=1}^{M} p_j(x)\,\bar{a}_{ij}\right) \quad (2)$$

where;

$u(x)$: the total displacement function.

N : the total number of original nodes before enrichment process.

$N_i(x)$: the standard shape function.

$\bar{u}_i$: the standard degree of freedom.

$\bar{N}_i(x)$: shape functions for the enrichment part.

$p_j(x)$: enrichment functions.

$\bar{a}_{ij}$: the new degree of freedom for the enriched nodes.

M : number of enrichment functions.

3. The Enrichment Functions

The basic concept of the enriching nodes in the XFEM is that employing the Partition of Unity (PU) concept for the improving of the finite element approximation.

Consider a domain Ω with a crack interface that divide the domain into (Ω^+, and Ω^-) as shown in **Fig. 1**. There are many enrichment functions that can be used for the crack interface problems one of the most popular enrichment function is the so-called the Heaviside enrichment function which was introduced by Moës[12].

The Heaviside function *H(x)*, can take two magnitudes; (0) if the solution approximation was on the negative part of the domain and (1) if the solution approximation was on the positive part of the domain.

$$H_{\Gamma d}(x) = \begin{cases} 1 & x \in \Omega^+ \\ 0 & x \in \Omega^- \end{cases} \quad (3)$$

then, the displacement solution will be:

$$u(x,t) = \hat{u}(x,t) + H_{\Gamma d}(x)\|u(x,t)\| \quad (4)$$

where; the symbol || || represent the perpendicular distance between *x*, and *t*.

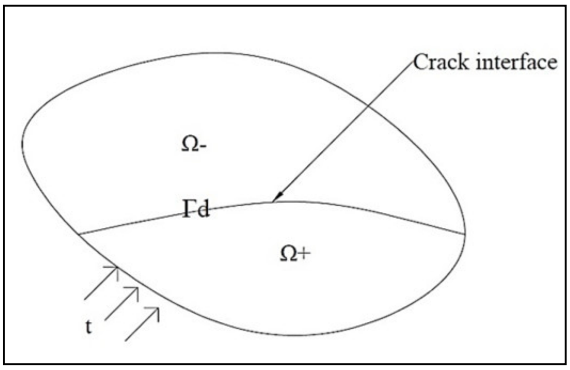


Figure 1: A domain with crack interface.

4. Concrete Beam Model

A simply supported plain concrete beam subjected to two-point load, was modeled as a two dimensional plan stress problem for the flexural behavior analysis. The dimensions of the beam were selected accordingto [5], which were 750 mm span, 150 mm width, and 150 mm depth. A small notch of 4 mm in depth was located at the center of the face of the specimen and the concrete numerical beam model. The loading case was relied onASTM C78-16[2]. **Fig. 2** illustrates the beam dimensions and loading state of the plain concrete beam model.

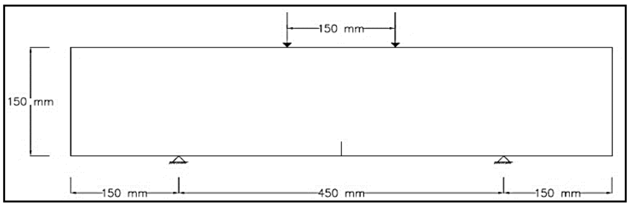


Figure 2: Dimensions, boundary conditions, and loading for the concrete beam model.

5. Experimental Work

Two sets of plain concrete beam specimens (B1, and B2) were prepared with the same BS standard dimensions (i.e. 150 mm length × 150 mm depth). Each specimen set consists of three similar beams and the average results were gained. Rounded coarse aggregatewas used with two different grain size distributions as shown in **Table 1** below:

Table 1: Coarse aggregate grading

Sieve size (mm)	Passing for specimen B1, %	Passing for specimen B2, %
37.50	100.00	100.00
19.00	93.86	100.00
9.50	52.80	90.70
4.75	5.74	10.67
2.36	0.00	0.05

The concrete mixes for both beam specimens were designed to give approximately the same compressive strength of 25 MPa. The quantities of concrete mix components for both beam specimens can be seen in **Table 2**.

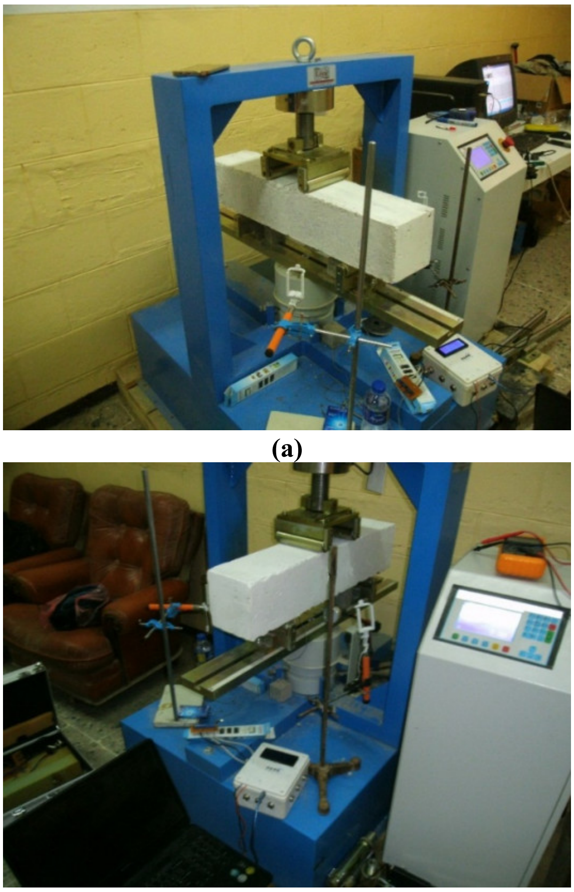
Table 2: Concrete mix quantities for the experimental work

Component	Mix for beam B1	Mix for beam B2
Cement, kg/m^3	340	350
Water, kg/m^3	205	210
Sand, kg/m^3	735	758
Coarse aggregate, kg/m^3	1015	943
Expected air voids content, %	2	3

Fig. 3 shows a part of the experimental work conducted for B1 and B2 specimens.

An electrical strain gage was fixed on the bottom face at approximately mid span of each specimen for strain measurementsin tension fibers. In addition, a dial gage was placed to measure the maximum deflection of the beam specimen during the loading stage.

A summary of the experimental resultsare listed in **Table 3** shown below, the results stated for the average value of three similar beams of the two models B1 and B2.



(a)

(b)

Figure 3: Loading system in the testing laboratory machine: (a) B1 specimen, and (b) B2 specimen.

Table 3: A summary of results from the experimental work

Results	Specimen B1	Specimen B2
Max. load, kN	28.31	16.75
Max. deflection, mm	1.093	1.007
Max. stain from stain gauge reading	$1.05\text{x}10^{-4}$	$1.04\text{x}10^{-4}$

6. Finite Element Model of the Concrete Beam

In this study, the plain concrete beam numerical model was constructed in the meso-scale feature. The concrete is considered as multi-phasic material consists of aggregate, cement mortar. The interfacial deformations between these two phases are neglected as the two materials were assumed to be fully bonded. The numerical model was constructed as a two dimensional plane stress problem. The elements size used was 4 mm for the whole model. Element type used in the numerical model was quadrilateral 8-node element.The total area of coarse aggregate in the model was computed from the concrete mix fractions as shown in **Table. 2**. The coarse aggregate fractions were divided according to the grading segments shown in **Table. 1**. The coordinates, orientation, and size within the limits of the grading segments were selected randomly by using EXCEL sheets for the mathematical calculations. Coarse aggregate particles were assumed to be elliptical in shape. The air voids were modeled as empty circle of 2 mm in diameter with the content percentage shown in **Table. 2**. ABAQUS 6.13 software was employed for the finite element modeling and analysis of the problem. The properties of cement mortar and coarse aggregate particles used in the numerical models that listed in **Table.4** were taken from [9]. The coarse aggregate and cement mortar were assumed to be linear elastic materials.

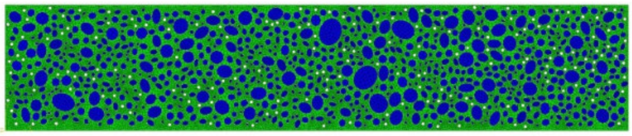
Table 4: Material propertiesLópez (2007).

Material	Elastic Modulus, E, MPa	Poisson's ratio	Fracture Energy, G_f, N-mm/mm^2	Maximum Tensile Strength, MPa
Aggregate	75000	0.2	-	-
Cement Mortar	25000	0.2	0.06	2.88

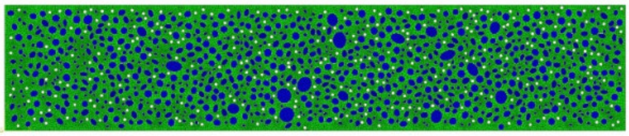
The maximum tensile strength of cement mortar shown in **Table 4**, represents the maximum tensile stress that concrete can handle before fracture, which its direction is perpendicular to the direction of the crack path.It was computed according to ACI 318-95[1], in which, the maximum direct tensile stress of concrete is assumed proportional to the square root of compressive strength of concrete$\sqrt{f_c'}$.

$$Max.\,Direct\ Tensile\ Stress = \left(0.43\sqrt{f_c'}\ to\ 0.71\sqrt{f_c'}\right) \quad (5)$$

The finite element models for B1, and B2, models are shown in **Fig 4**.



(a)



(b)

Figure 4: Finite element meso-scale model of concrete beam: (a) for model B1, and (b) for model B2.

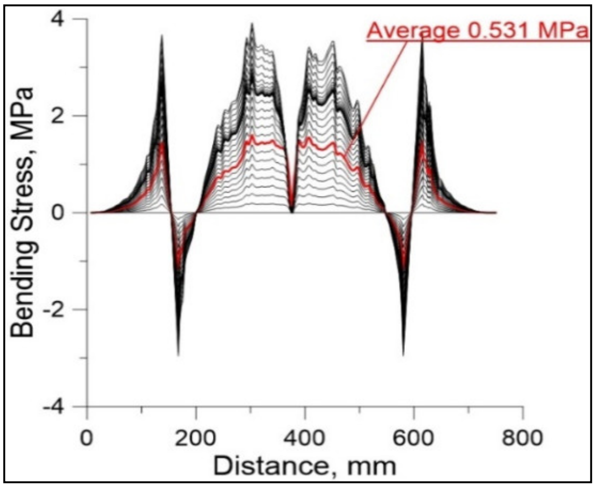
7. Results and Discussion

The finite elements models of the plain concrete beams shown in previous section were subjected to two point loading case as discussed in section 3. It was found that the maximum applied load is approximately the same of fracture load of the specimens in the experimental work. In meso-scale FE model the effect of the non-homogeneity of the concrete material was obviously observed. The bending stress distribution along the bottom fiber of the models is shown in **Fig. 5.** The distribution of shear stress along the depth at 225 mm from the end support is shown in **Fig 6.**

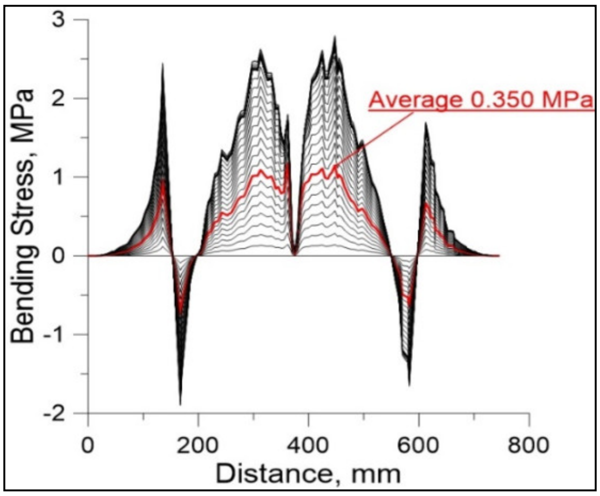
The red lines shown in **Fig 5** and **Fig 6** represent the bending and shear stresses at the first initiation of crack. The numerical analysis shows that for beam model B1 the crack is initiated at a load of 11.05 kN, while in beam model B2 the crack initiation load is approximately 7 kN. **Fig 5.** shows that the bending stress of the beam model approaches to zero at the mid span of the beam, this indicates that the developed crack is working as a plastic hinge through which, the beam is no longer transfer moment. The sinuous shape of the bending and shear stress may be attributed by the existing of the coarse aggregate particles and the air voids near the bottom fiber of the beam.

Besides, it is noted that, the average value of the bending stress at crack initiation for model B1 is bigger than that of model B2. This finding is supported by the experimental results in**Table 3** in which the maximum applied fracture load for model B1 is bigger than that of model B2 this is may be due to the coarser aggregate particles used in construction of model B1. This is also noted inthe shear stress distribution shown in **Fig 6** which indicates that the meso-scale numerical model of concrete beam B1 requires a bigger load than concrete beam B2 for the fracture process. Experimentally this was noticed via recording a maximum applied load for B1 bigger than for B2.

On the other hand, the maximum measured deflection in specimen B1 was found bigger than B2 by 11 % at the same applied load. This is may be due to the existing of larger coarse aggregate particles in specimen B1 and the expected air voids content in specimen B2 is more by 1% than beam B1 which facilitate the concrete beam to be fractured at lower applied load.



(a)



(b)

Figure 5: Bending stress diagram with respect to the span length of the plain concrete beam: (a) for model B1, and (b) for model B2.

Fig. 7 and **Fig. 8** show the relation between applied force and strain at mid span of the bottom face of the two beam models in experimental and numerical analysis. Experimentally, the strain was measured by the method of electric strain gauges attached at the mid span of the bottom face of the specimen. In addition, the strain at the same location was calculated assuming the concrete is elastic-isotropic-homogenous material using the well known Euler-Bernoulli beam theory by applying Equation (6).

$$\epsilon_{Theoretical} = \frac{M.c}{E.I} \qquad (6)$$

Where,

M: the applied moment.

E: modulus of elasticity of concrete, $E = 4700\sqrt{f_c'}$

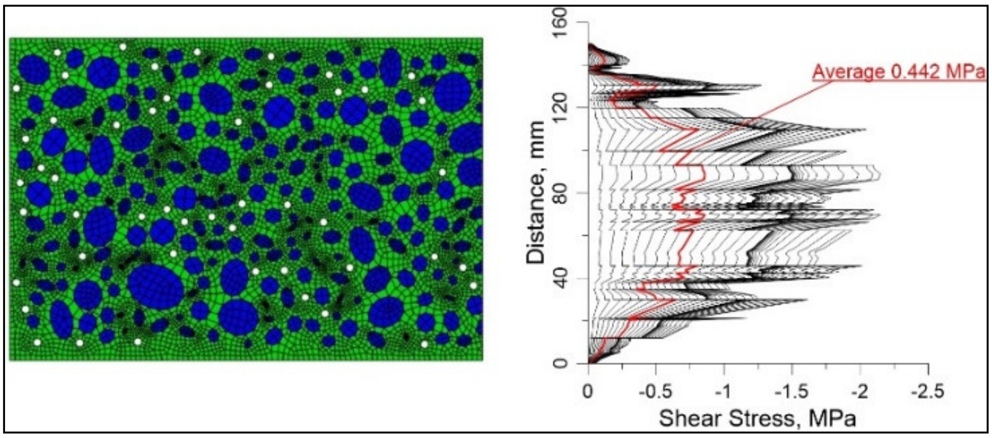
c: the distance from the bottom fiber to the neutral axis of the beam.

I: moment of inertia of the cross section of beam.

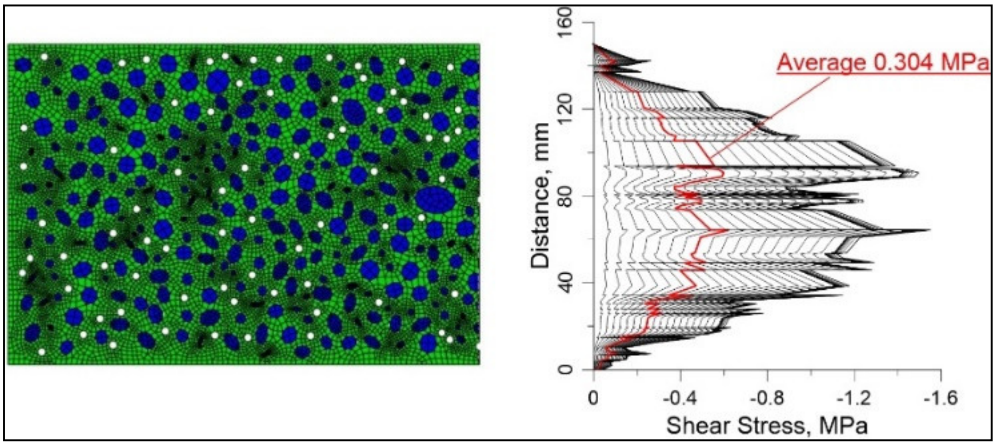
It can be seen from **Fig. 7** and **Fig. 8** that the convergence between experimental, numerical, and theoretical results is more obvious in **Fig. 8** than **Fig. 7.** This finding can be attributed to the fact that says that the homogeneity of concrete is increased with decreasing of the maximum size of aggregate. Furthermore, this convergence indicates the validity and powerful of Extended Finite Element Method (XFEM) in fracture analysis of concrete members.

Crack propagation paths in concrete beam B1 and B2 in both experimental work and numerical analysis can be seen in **Fig. 9**. It is clear that the predicted paths have almost similar shape of those in experimental specimens. The reason stand behind similarities in these paths is the existence of coarse aggregate particles which have a tensile strength more than cement mortar. Consequently, the path of crack propagation passes through cement mortar only.

It is realistic to concludethat when the aggregate particles goes coarser the crack path shape approaches a zigzag line while if the aggregate is finer, the path will be more like straight line. It is though that the zigzag crack path in the coarser aggregate model B1 gave the additional flexural strength than the finer aggregate model B2.



(a)



(b)

Figure 6: Shear stress diagram with respect to the cross section of the beam at 225 mm from the left hand support: (a) for model B1, and (b) for model B2.

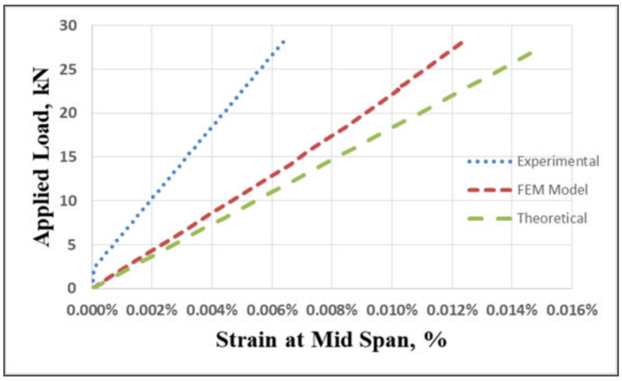


Figure 7: Applied load versus the strain for the numerical analysis, theoretical calculations, and the experimental results for model B1.

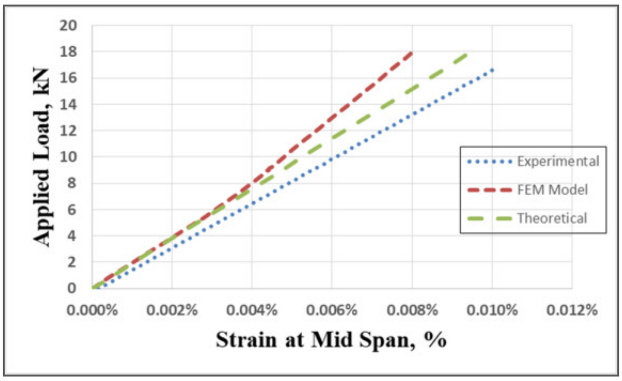


Figure 8: Applied load versus the strain for the numerical analysis, theoretical calculations, and the experimental results for model B2.

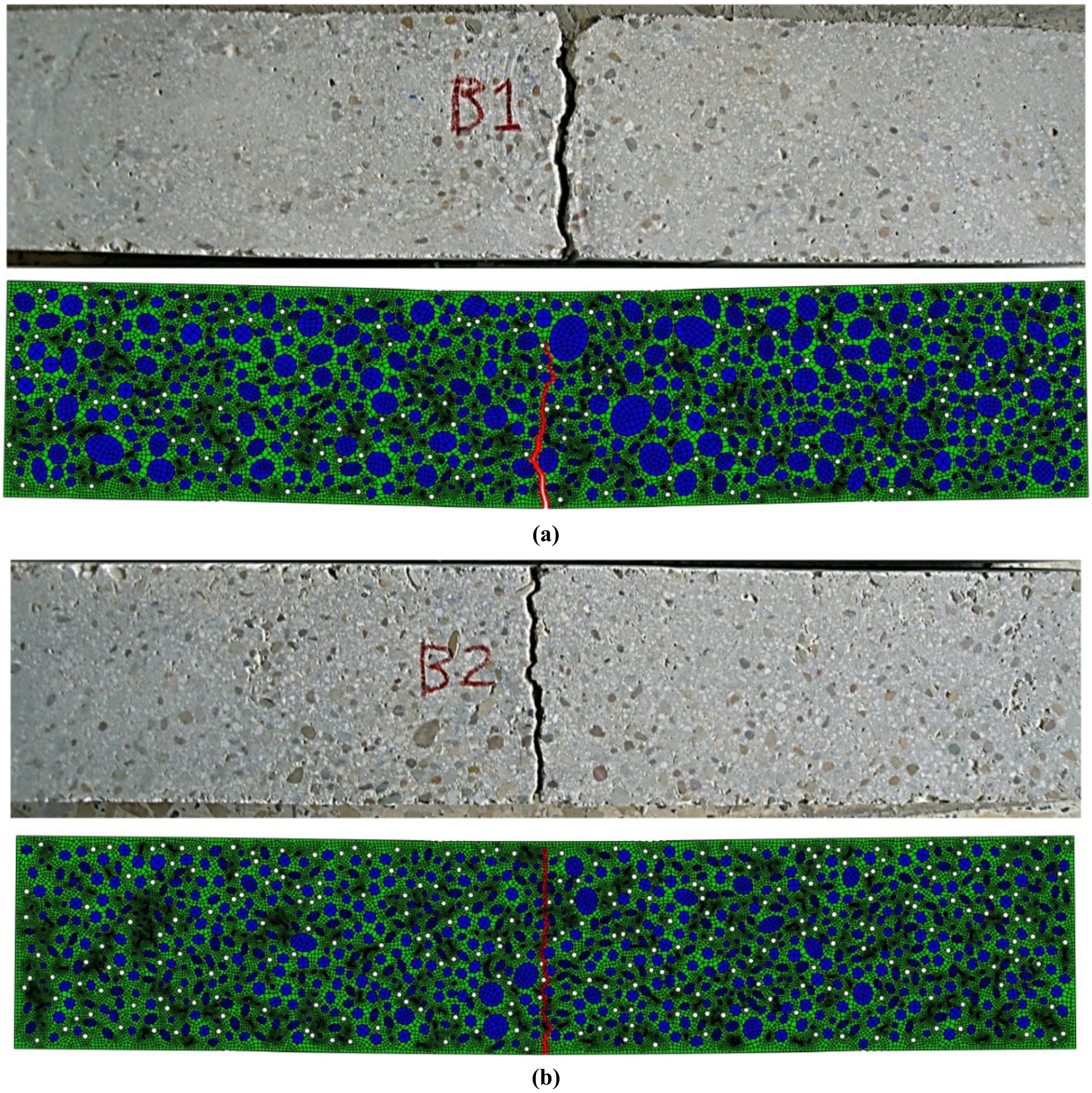


Figure 9: Crack Propagation paths and coarse aggregate particles distribution for the numerical and experimental concrete models: (a) for model B1, and (b) for model B2.

8. Conclusion

The following conclusions were drawn from the experimental work and FE numerical analysis results:

- The meso-scale FE analysis approach gives better understanding of the behavior of non-homogenous materials such as concrete, which is consisting of two materials mortar and aggregate.
- The Extended Finite Element Method (XFEM) was found a powerful solution for the discontinuity problems that faced during the fracture process of concrete members.
- The distribution of the shear stress on the cross section of the concrete beam is not uniform since, itwas affected by the exiting of the coarse aggregate particles and the entrapped air voids into concrete media.
- The existing of the air voids leads to the stress concentration problems that affect the trend of the bending stress diagram of the plain concrete beam.
- The crack propagation paths are directly affected by the existence of coarse aggregate since it passes through the cement mortar only.

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سلوك الخرسانة عديمة التسليح وتحليلها باستخدام طريقة العناصر المحددة الموسعة

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الخلاصة – تم تحليل اجهادات الانحناء فياعتاب خرسانية عديمة التسليح في حاله اسناد بسيطة وتحميلها بنقطتين من أجل الثني. تم نمذجة الاعتاب عدديا باستخدام نموذج المتوسط المدى للخرسانة وتم اعتبار الخرسانة مادة ثنائية الطور تتكون من مادتين (مونة السمنت , والركام).وقد تم التحري عن فشل الخرسانة في المختبر وفي النموذج العددي عند تعرضها الى اجهادات انحناء.تم استخدام طريقة العناصر المحدودة الموسعة (XFEM) لحل مشاكل اللاستمرارية التي تظهر أثناء عملية الكسر في الخرسانة. تم استخدام برنامج ABAQUS مع استعمال خاصية standard/explicitly للتحليل العددي. حبيبات الركام الخشن تم افتراضها ذات اشكال اهليجية.تم أخذ بنظر الاعتبار خصائص أخرى في الركام الخشن مثل احجام وقياسات حبيبات الركام التي تم قياسها من الفحوصات المختبرية التي اجريت على الاعتاب الخرسانية.تم استعمال عتبين من اجل التحليل العددي والمختبري.تم تركيز الفرق بين الاعتاب في المقاس الأقصى لحبيبات الركام الخشن.أظهرت المقارنة بين النتائج التجريبية والعددية أن نموذج المتوسط المدى يعطي واجهة جيدة لتمثيل النماذج الخرسانية في التحليل العددي. وقد استنتج أن XFEM هو تقنية قوية تستخدم لتحليل عملية الكسر وانتشار التشققات في الخرسانة..

الكلمات الرئيسية – طريقة العناصر المحددة الموسعة، مقاومة الانحناء للخرسانة، ميكانيكا الانكسار، النمذجة المتوسطة المدى، تحميل نقطتين.