



# Formulation of Camshaft Dynamic Load Factor under Uniform Acceleration Excitation

Safa Ali Mahmood shukri <sup>1</sup> and Nassear Rashied Hamoad <sup>2</sup>

<sup>1</sup> Department of Mechanical Engineering, University of Baghdad, Baghdad, Iraq, [safa.shukri2003m@coeng.uobaghdad.edu.iq](mailto:safa.shukri2003m@coeng.uobaghdad.edu.iq)

<sup>2</sup> Department of Mechanical Engineering, University of Baghdad, Baghdad, [nassear\\_machine@yahoo.com](mailto:nassear_machine@yahoo.com)

\*Corresponding author: Safa Ali Mahmood shukri, email [safa.shukri2003m@coeng.uobaghdad.edu.iq](mailto:safa.shukri2003m@coeng.uobaghdad.edu.iq)

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**Abstract—** Due to their importance in many mechanical applications, Cams was the subject of many researches regarding the improvement strength to withstand dynamic contact stresses and wear based on cam geometry modifications and follower variables. This work assesses the effect of uniform acceleration cam variables on the induced dynamic forces and camshaft dynamic load factor. The equation of motion is formulated based on Newton's second law taking into account the geometry of cam lobe and follower motion to evaluate the dynamic contact force during the course of action in terms of camshaft angular speed and time. Theoretical analysis contains two parts these are; the evaluation of static and dynamic contact stresses based on Hertz and dynamic load factor formulation for single element camshaft as a continuous simply supported beam based on Euler- Bernoulli beam theory. FEM has been adopted to simulate the static and dynamic contact stresses and dynamic deflection for multi elements camshaft and to verify the result of theoretical solution. Results show that the increasing of damping ration and spring stiffness has appositve effect on dynamic load factor reduction while the increasing of follower mass has negative effect on dynamic load factor reduction.

**Keywords—** camshaft, uniform acceleration motion, dynamic load factor.

## 1. Introduction

Camshaft is crucial component in modern internal combustion engines, responsible for controlling the opening and closing of the engine's valves. It was first described by engineer Al-Jazari in 1206, who used it in his automata, water-raising machines, and water clocks such as the castle clock [13]. Today, camshafts are made from high-strength materials like steel or aluminum and are designed with precision to ensure optimal engine performance. They are an essential part of any combustion engine, providing precise timing for fuel injection and exhaust valve operation. The design of camshafts has evolved over time, with modern engines using variable valve timing technology to further optimize performance and fuel efficiency. Despite its ancient origins, the camshaft remains a vital part of modern engineering and continues to play a critical role in powering everything from cars to heavy machinery. Camshaft consist from cams that distributed along it. The cam is attached to a follower, which moves up and down as the cam rotates, creating an oscillating motion. Cams can be classified into three main groups based on their shape: wedge, plate and

cylindrical [1]. Another classification system categorizes cams into groups based on the motion of the follower. These groups include cams that produce cycloidal motion, simple harmonic motion, uniform velocity and uniform acceleration. Cams can also be categorized according to the follower's design, including knife-edge, roller, flat, and spherical face followers.

Due to the operating conditions that cams are often subjected to, including high loads, temperature fluctuations, and pressure changes, there has been significant research into the development of cams that are able to withstand these conditions and maintain their performance and lifespan. Researchers have studied various factors that can impact the performance and lifespan of cams, including kinematic behavior, contact

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Stress endurance, wear resistance, vibration, heat generation, and noise emission, and have developed techniques for designing cams that are able to withstand these conditions. The smooth, oscillating motion of the follower produced by a harmonic cam can help to reduce the noise and wear that are typically associated with cam and follower systems. Uniform acceleration cam (parabolic cams) are cams that produce parabolic motion in the follower, which means that the follower moves in a path that follows a parabolic curve. Parabolic cams are often used in applications where high acceleration or deceleration is required, such as in aircraft or automotive systems and it used to control linkages in complex machinery [8].

Cams are used in wire-forming machinery, packaging machines, mechanical and electronic computers and internal combustion engines [4].

There were many works related to the current work that could be followed as: Desai and Patel [9] evaluated the critical speed of rotation to avoid jamping based on dynamic analysis for different cam profiles in a addition to that contact stress has been analyzed too. The study shows that the resultant force that act on the cam must has negative value i.e., the force acts downward that follower maintain contact with cam until reach the critical speed. Dinesh and Kalita [5] utilized FE Analysis of Tangent Cam for Different Positions of the Roller Follower to evaluate contact stresses between the cam and follower. The effect of increasing angular speed on cam surface has been explain. Von misses stresses has been found out and plotted at all the positions comparatively. Result show that when cam lift the follower at it's highest position i.e. maximum lift, the stresses are maximum., these stresses at maximum lift at high speed cam rotating are very dangerous which cam start a crack at the surface of the cam and propagate further giving adverse effect on the cam. Aveen [1] studied the induced contact stress in simple harmonic cam with roller follower under different design parameters (cam face width, follower radius rise and return angles, and modulus of elasticity), theoretically using Hertz contact theory and FEM while the experimental work adopted photoelastic stress investigation method. Results show that reducing of modulus of elasticity has appositve effect to reduce contact failure. Increasing cam width has appositve effect to reduce contact failure. Increasing cam width has

Manoj et als [10] found the stress analysis that carried out the snail cam by analytical, FEM and photoelasticity technique. From analytical result observed that the maximum stress in cam occur at downward stroke of follower where follower strikes on cam with return stroke. The maximum stress accrued from experimental

Analysis less than accrued from finite element method about (3.8) %.

Nassear at als [11] focused on determining the Dynamic Load Factor (DLF) for a single-element camshaft under harmonic excitation. The research involved analyzing both static and dynamic contact stresses that occur throughout the cam's motion. These stresses were calculated using Newton's second law and incorporated into the Hertz equation to assess the maximum induced dynamic and static contact stresses. To validate the accuracy of the stress results, Finite Element Method (FEM) analysis was conducted using the Ansys solver version 15. This analysis confirmed that the Hertz equation remains reliable for predicting cam stress levels. The dynamic deflection of the camshaft during steady- state motion was determined by solving a non- homogeneous second-order ordinary differential equation. This deflection was then utilized to compute the Dynamic Load factor.

Dynamic load factor has been evaluated for different damping ratio and different frequency ration. The result show that the maximum static force and stress acting on the maximum cam lift (stroke) while the dynamic contact force and stress located at start angle of rotation. The results also show that dynamic load factor has peak value at frequency ratio 1, furthermore, augmenting the damping ratio significantly reduces the dynamic load

Factor near the frequency ratio  $\xi = 1$ . The contour of the

Cam also impacts contact stresses in dual manners,

Regulating both the active dynamic force and the curvature radius. These impacts, however, exhibit contradictory effects.

This work analyzes the static and dynamic contact stresses of the uniform acceleration motion cam during the whole course of action while the second aim is to find the dynamic load factor for the camshaft to be used in the static analysis of the camshaft dynamic performance.

## 2. Cam profile:

The key factor of any camshaft are the lobes which is the force manipulation element to be accurately represented. Uniform acceleration cam-follower system with roller contact mechanism, Kinematic analysis involves the calculations of displacement, velocity and acceleration of the follower at different instant. When the velocity increases at a uniform rate during the first half of the motion follower and decreases at a uniform rate during the second half of the motion. The acceleration is constant and positive throughout the first half of the motion, and is constant and negative throughout the second half. This type of motion gives the follower the smallest value of maximum acceleration along the path of motion. The key factor of any camshaft are the lobes which is the force manipulation element to be accurately represented. This work spots the light on the harmonic cam-follower system with roller contact mechanism, Kinematic analysis involves the calculations of displacement, velocity and

acceleration of the follower at different as following. It is well known that the kinematic components for uniform motion cam system are [7]

When  $0 \leq \theta \leq \beta$

$$r(\theta) = r_b + 2s \left(\frac{\theta}{\beta}\right)^2 \quad (1)$$

$$\dot{r}(\theta) = 4s \left(\frac{\omega t}{\beta}\right)^2 \quad (2)$$

$$\ddot{r}(\theta) = 4s \left(\frac{\omega}{\beta}\right)^2 \quad (3)$$

$$\ddot{r}(\theta) = 4s \left(\frac{\omega}{\beta}\right)^2 \quad (4)$$

When  $(\beta/2) < \theta < \beta$  kinematic components will be:

$$r(\theta) = r_b + s \left[1 - 2\left(1 - \frac{\theta}{\beta}\right)^2\right] \quad (5)$$

$$\dot{r}(\theta) = 4s \left(\frac{\omega}{\beta}\right) \left(1 - \frac{\theta}{\beta}\right) \quad (6)$$

$$\ddot{r}(\theta) = -4s \left(\frac{\omega}{\beta}\right)^2 \quad (7)$$

$$\ddot{r}(\theta) = -4s \left(\frac{\omega}{\beta}\right)^2 \quad (8)$$

Where  $r(\theta)$  is the follower displacement and  $\dot{r}(\theta)$  is the follower velocity,  $\ddot{r}(\theta)$  is the follower acceleration,  $r_b$  is the base radius of cam,  $S$  is the maximum stroke,  $\beta$

is the raise and return angle,  $\theta$  is the angular position (degree) and  $\omega$  is the angular speed.

### 3. Contact force:

During the course of action cam and follower interaction arises variable contact force which puts the system to work under indispensable vibration condition to be formulated and assessed for further enhancement. Static contact force has been satisfied from spring stiffness with follower displacement (cam radius of curvature) the equation that describe static contact stress is:

$$F_0 = S(\theta) K_s \quad (9)$$

Dynamic contact force formulation is the dominant factor regarding the control of the induced stresses, wear, heat and noise generation, and noise generation as well as the camshaft deflection during the design process. Starting from Newt's second law equation of motion is derived for uniform acceleration cam to evaluate the dynamic contact forces with some assumptions such as there is no relative sliding between the roller and cam and in turn no friction force, follower works with no damping as follows:

$$F_d = m_f \ddot{r}(\theta) + k_s [r(\theta) - rb] \quad (10)$$

$$\text{When } 0 < \theta < \left(\frac{\beta}{2}\right)$$

By substitute equation (1), (4) in to equation (10) to get:

$$F_d = 4 m_f s \left(\frac{\omega}{\beta}\right)^2 + 2k_s s \left(\frac{\theta}{\beta}\right)^2 \quad (11)$$

$$\omega/\beta = \Omega \quad (12)$$

$$F_d = 4 m_f s \Omega^2 + 2s k_s \Omega^2 t^2 \quad (13)$$

$$\text{For } \left(\frac{\beta}{2}\right) \leq \theta \leq \beta$$

By substitute equation (5), (8) in to equation (10) to get:

$$F_d = -4 m_f s \left(\frac{\omega}{\beta}\right)^2 + k_s \left[1 - 2\left(1 - \frac{\theta}{\beta}\right)^2\right] \quad (14)$$

$$F_d = -4 m_f s \Omega^2 - s k_s + 4s k_s \Omega t - 2s k_s \Omega^2 t^2 \quad (15)$$

Where  $F_d$  is the dynamic contact force,  $m_f$  is follower mass,  $k_s$  stiffness of spring.

Equations (11),(14), propose the dynamic induced contact forces for uniform acceleration motion in terms of follower mass, spring stiffness, stroke, and the operating angular speed.

In this work, camshaft was considered as a continuous simply supported beam subjected to transverse vibration with no account to the unbalance forces of cam elements.

To compute deflection equation must be first knowing the natural frequency and the mode shape using Euler-Bernoulli equation, this is a fundamental equation in the theory of beams, which describes the behavior of a beam under bending. It states that the deflection ( $y$ ) of a beam at any point along its length is proportional to the integral of the bending moment ( $M$ ) along the beam's length, up to that point. Euler- Bernoulli equation for damped shaft is [3]:

$$EI \frac{\partial^4 y}{\partial x^4} + m \frac{\partial^2 y}{\partial t^2} + c \frac{\partial y}{\partial t} = 0 \quad (16)$$

$m$ ,  $c$  are the mass and viscose damping constant per units length respectively

$E$  is the modulus of elasticity,  $(I)$ : is the second moment of area.

To solve the above P.D.E by assume:

$$y(x, t) = \Phi(x) * q_n(t) \quad (17)$$

By substitute equation (17) to equation (16) and solving equation get [10]:

$$\omega_n = (n\pi)^2 * \sqrt{\frac{EI}{m l^4}} \quad (18)$$

$$\Phi(x) = \sum_1^\infty \sin\left(\frac{n\pi}{l} x\right) \quad (19)$$

**Where:**  $\Phi(x)$  is the mode shape for nth mode and  $\omega_n$  natural frequency.

To compute the deflection of the shaft for single for act on mid span of shaft beam at  $x=(L/2)$  the equation of motion is [10]:

$$\ddot{q}_n(t) + 2\zeta\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = \frac{F_d^* \Phi_n(x)}{m \int_0^L \Phi_n^2(x) dx} \quad (20)$$

Where:

$$\frac{F_d^* \Phi_n(x)}{m \int_0^L \Phi_n^2(x) dx} = \frac{4 m_f s \Omega^2 + 2 s k_s \Omega^2 t^2 + \sum_1^{\infty} \sin \frac{n\pi}{2}}{m \frac{L}{2}} = Q(t) \quad (21)$$

$$\xi = \Omega \omega_n$$

Q: is the normalized force acting on the middle of beam at  $x=L/2$

When  $0 < \theta < (\frac{\beta}{2})$  for  $n=1, x=L/2$  the solution of equation (20) is:

$$q_n(t) = \frac{2s k_s}{m\omega_n^2 l} [At^2 + Bt + D] \quad (22)$$

Where:

$q_n(t)$  is generalized coordinates response

$$A = 2 \omega_n^2 \xi^2$$

$$B = -8\zeta \eta \xi^2 \omega_n$$

$$D = \left( \frac{4m_f \omega_n^2 \eta^2}{k_s} + 16\zeta^2 \xi^2 - 4\xi^2 \right)$$

The maximum deflection of camshaft will be gate by substitute (22) and (19) in (17) as below:

$$y(L/2, t) = [At^2 + Bt + D] \quad (23)$$

$$\text{Where } \frac{2s k_s}{m\omega_n^2 l} = \frac{s k_s}{k}$$

When  $\beta/2 < \theta < \beta$  for  $n=1, x=L/2$

$$\ddot{q}_n(t) + 2\zeta\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = \frac{F_d^* \Phi_n(x)}{m \int_0^L \Phi_n^2(x) dx}$$

$$F_d = -4 m_f s \Omega^2 - s k_s + 4 s k_s \Omega t - 2 s k_s \Omega^2 t^2$$

$$q(t) = \frac{2s k_s}{m\omega_n^2 l} [Gt^2 + Jt + V] \quad (24)$$

Where:

$$G = -2 (\xi\omega_n)^2$$

$$J = (4\xi\omega_n + 8\zeta \xi^2 \omega_n)$$

$$V = (4 \frac{mf}{k_s} (\xi\omega_n)^2 + 4\xi^2 - 8\xi\zeta - 1)$$

By substitute (24) and (19) in (17)

Get:

$$y(L/2, t) = \frac{s k_s}{k} [Gt^2 + Jt + V] \sum_1^{\infty} \sin(\frac{\pi}{2}) \quad (25)$$

The maximum static deflection of the camshaft due to the follower spring force is:

$$y_0 = \frac{F_0}{K} = \frac{s k_s}{K} \quad (26)$$

Where  $k$  is the stiffness of shaft and its value ( $k = 48 EI / L^3$ ) under simply supported conditions.

#### 4. Dynamic Load Factor (DLF)

It takes a lot of time to calculate the dynamic performance of the structures in addition to the enormous effort to find its calculations represented by the stress and deflection, so the so-called dynamic load factor was found, which provides ease of finding the dynamic performance from the static analysis. Dynamic load factor (DLF) of camshaft is a measure of the fluctuating loads that a camshaft experiences during its operation. It is typically defined as the ratio of the peak dynamic load on the camshaft to the static load on the camshaft, and is used to determine the fatigue strength of the camshaft under these dynamic loading conditions.

Dynamic load factor has been used in many applications, the most important of which are bridges, as well as there are many researches and studies. The dynamic load coefficient for mechanical parts such as gears was found and calculated. On mechanical system dynamic load factor range between 1 to 2 which is related to sudden applied load, however in other applications, depending on the natural frequency damping ratio and the natural length, it may reach 10 or more. Theoretically (DLF) can be obtained from the ratio of dynamic to Max. Static deflection or the ratio of dynamic stresses to Max. Static stress. The equations that describe dynamic load factor is [12]:

$$DLF = \frac{y(x,t)}{y_0} = \frac{\sigma}{\sigma_0} \quad (27)$$

Where  $y_0$  and  $y$  are static and dynamic deflection respectively,  $\sigma_d$ , is the maximum dynamic and static stress.

Dynamic load factor for interval  $(0 < \theta < (\beta/2))$  will be obtained by substitute (23) and (26) in (27) to get:

$$DLF = [At^2 + Bt + D] \quad (28)$$

$$A = 2 \omega_n^2 \xi^2$$

$$B = -8\zeta \eta \xi^2 \omega_n$$

$$D = \left( \frac{4m_f \omega_n^2 \xi^2}{k_s} + 16\zeta^2 \xi^2 - 4\xi^2 \right)$$

Substitute (25) and (26) in (27) to evaluate Dynamic load factor for interval  $\beta/2 \leq \theta \leq \beta$  has:

$$DLF = [G t^2 + J t + V] \quad (29)$$

$$G = -2 (\xi \omega_n)^2$$

$$J = (4\xi \omega_n + 8\zeta \xi^2 \omega_n)$$

$$V = \left( 4 \frac{mf}{k_s} (\xi \omega_n)^2 + 4\xi^2 - 8\xi\zeta - 1 \right)$$

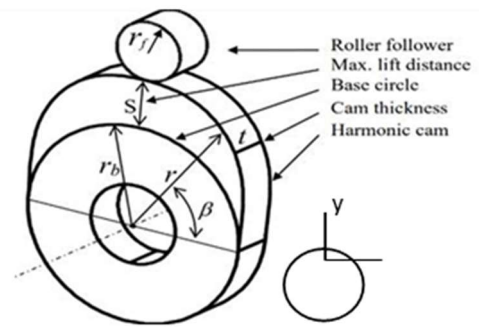
The equations (28), (29) propose the dynamic load factor in term of damping ratio (natural frequency ( $\omega_n$ ) and frequency ratio ( $\xi$ ).

The cam is dealt with as a concentrated load on simply supported beam regarding the stiffness idealization, the material is structural one with isotropic linear elastic behavior. **Table 1:** studied cases geometrical and mechanical properties properties. **Figure. 1** Cam shape and terminology.

**Table 1:** studied cases geometrical and mechanical properties properties.

$r_{sh}$	15mm
Base radius( $r_b$ )	30mm
Follower radius ( $r_f$ )	10mm
Maximum lift stroke S (mm)	10mm
Camshaft length (L)	1000 mm
(Cam width b)	10mm
Rise and return angle ( $\beta$ )	80
Cam radius ( $r_c$ )	40 mm
Camshaft mass (m)	1 kg/m

Follower mass(mf)	0.1
Natural frequency ( $\omega_n$ )	879 rad/sec
Camshaft stiffness (k)	381 N/mm
Spring stiffness ( $k_s$ )	10 N/mm
Modulus of elasticity E	200000 Mpa
I	39740.6 mm <sup>4</sup>
Poisson ratio ( $\nu$ )	0.3



**Figure 1:** Cam shape and terminology [8].

## 5. Stress Analysis

Stress analysis of a cam and follower system evaluates the induced contact stresses. This can be useful for understanding how cam and follower will behave under different operating conditions and identifying potential areas of stress concentration or failure.

There are several approaches that can be used to evaluate stress analysis of a cam and follower system and the most in use approach is Hertz equation. Hertz equation is a mathematical formula that describes the relationship between the forces acting on a pair of bodies in contact with each other in term of the resulting Max. Contact stress and deformation area dimensions.

Hertz equation can be used to design a cam and follower system that will operate reliably and efficiently under a given set of conditions. It can also be used to optimize the design of the system for specific performance characteristics, such as reducing wear or minimizing deformation.

In the case of a cam and follower system, the Hertz equation can be used to calculate the contact stress between the cam and follower as the cam rotates and the follower rides on its surface. The equation takes into account factors such as the materials of the cam and follower, the radius of the cam, the load on the follower. Where [11]:

$$\sigma_{C_0} = \sqrt{\frac{F k_s * \left(\frac{1}{r} + \frac{1}{r_f}\right)}{\pi b \left(\frac{1-\nu_c^2}{E_c} + \frac{1-\nu_f^2}{E_f}\right)}} \quad (30)$$

Where  $\sigma_C$  is the contact stress, F is the applied load, b is the width of the cam,  $r_c$  and  $r_f$  are the cam and the follower radius of curvature at the contact place,  $E_c$  and  $E_f$  are the modulus of elasticity for the cam and follower.

For static contact force is:

$$\sigma_{C_0} = \sqrt{\frac{(r-r_b) k_s * \left(\frac{1}{r} + \frac{1}{r_f}\right)}{\pi b \left(\frac{1-\nu_c^2}{E_c} + \frac{1-\nu_f^2}{E_f}\right)}} \quad (31)$$

Dynamic contact stress for uniform acceleration cam getting by substituted equations (1) and (11) in to (30) for interval (0 to  $\beta/2$ ) is:

$$\sigma = \sqrt{\frac{4 m_f s \left(\frac{\omega}{\beta}\right)^2 + 2 k_s s \left(\frac{\theta}{\beta}\right)^2 * \left(\frac{1}{r} + \frac{1}{r_f}\right)}{\pi b \left(\frac{1-\nu_c^2}{E_c} + \frac{1-\nu_f^2}{E_f}\right)}} \quad (32)$$

For interval ( $\beta/2$ )  $< \theta < \beta$  Dynamic load factor is:

$$\sigma = \sqrt{\frac{[-4s m_f \left(\frac{\omega}{\beta}\right)^2 + k_s s [1-2\left(1-\frac{\theta}{\beta}\right)^2] \left[\frac{1}{r_b+s[1-2\left(1-\frac{\theta}{\beta}\right)^2]} + \frac{1}{r_f}\right]]}{\pi b \left(\frac{1-\nu_c^2}{E_c} + \frac{1-\nu_f^2}{E_f}\right)}} \quad (33)$$

The equation of (32), (33) represent dynamic contact stresses in term of cam radius, follower radius, Poisson ratio, modulus of elasticity and dynamic contact uniform acceleration cam.

## 6- Finite Element Method

The finite element method is one of the most extensively accepted and utilized tools for solving linear and nonlinear partial differential equations that emerge during mathematical modelling of diverse processes [6]. Large structures, extreme and complex working conditions, and lengthy solutions are the most significant challenges confronting researchers and engineers during the design

process of innovative or modified products, with finite element solution serving as the in-hand power tool for easy, cheap, and accurate solutions [2]. Finite element has been used also to investigate the analytical result of static and dynamic performance.

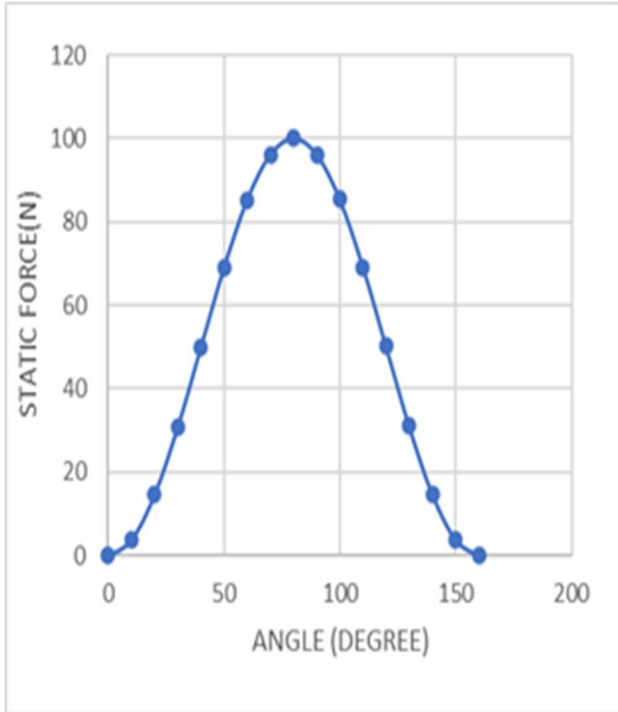
For static investigation the engineering representation of cam and follower geometry were basic step to make experimental and numerical part of stress analysis according to equation (1) That included the displacement of follower (r) as a function angle of rotation of cam its can be represented using Quick BASIC language. This data will be saved as xtl file and feed it to Ansys to be processed and used in numerical analysis in addition to build up of FE models and verify the contact stress values, ANSYS APDL (ver.21) was used. Studied case are structural material linear behaviour with 200 Gpa modulus of elasticity and 0.3 Poisson ratio as a mechanical property for the investigation of static contact stress. The follower was modelled as a cylinder. A convergence test was performed to determine the best type of element to use for the cam shape, and it was found that the brick solid element with 20 nodes was the best fit. The contact stress between the cam and the follower was studied using frictionless contact with two different elements, with the cam represented by element 174 and the follower represented by element 170. Both the cam and the follower were assigned the same mechanical properties, which were structural, linear, elastic, and isotropic with a Young's modulus of 200 GPa and Poisson's ratio of 0.3. A load of 100 N was applied in the negative y direction to the center of the follower at an angle of 80 degrees (when the cam's radius was 40 mm). The cam was free to rotate about its center, while the follower was free to both rotate about its center and translate in the y direction. Static investigation done by Ansys APDL to investigate contact stress obey Hertz equation formula. Dynamic deflection has been investigated using Ansys work bench solver in order to checking the validity of equations (23), shaft was drawn using Ansys design model, meshed and select the boundary condition fixed support on lift end and displacement support in the other end. The deflections have been found for single element camshaft and for multi elements cam. Force data has been taken according to equation (15) by programing this equation on Excel table sheet subjected these forces in many situations.i.e., shaft subjected to single force on mid span for first case and four forces for other cases (the shaft part of four stroke IC engine), Taking into consideration the phase difference angle of force distribution.

## 7. RESULTS AND DISCUSSION

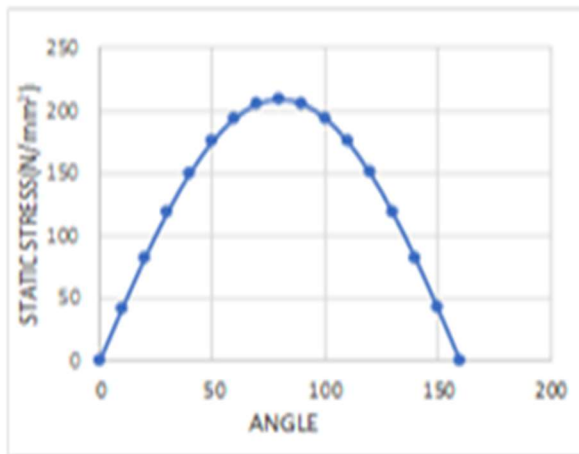
### 7.1 Static and dynamic contact force and stress

Depending on the mathematical equations of static force will be calculated according to equation (9) different contact positions during the course of action. It is clear that, the maximum static force value is 100N and take place at angle  $\theta = 80$ . As shown **Figure 2**. Static contact force has been adopted according equation (9). **Figure 3**

indicates the static contact stress for rise and return decent course, it's clear that, the maximum static contact stress value is (209.1556) N/mm<sup>2</sup> and take place at  $\theta= 40$ .

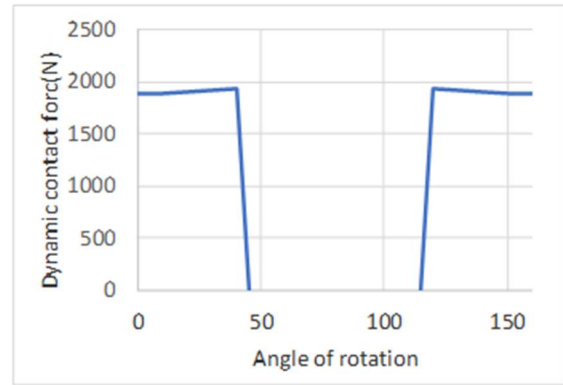


**Figure 2:** Spring force variation during the rise and decent courses.



**Figure 3:** Static contact stress during the rise and decent course

Uniform acceleration cam motion Dynamic contact force indicated according to equations (11) for interval (0 to 40) and (14) for interval (40 to 80) and it has maximum value at  $\theta=40$ , and equal 1931 N as **Figure4**. While maximum contact stress located at start angle of rotation at  $\theta=0$  and has a value of 936 N/mm<sup>2</sup>.



**Figure 4:** Dynamic contact stress during the rise and decent course.

The effect of different geometrical parameter has been examined to weigh their effect on contact force and contact stress as following:

1. Rise and return angle  $\beta$

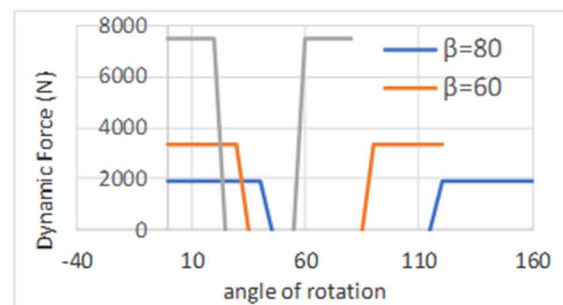
The effect of rise and return angle on contact force and contact stress will be discussed as follows:

• Dynamic contact force

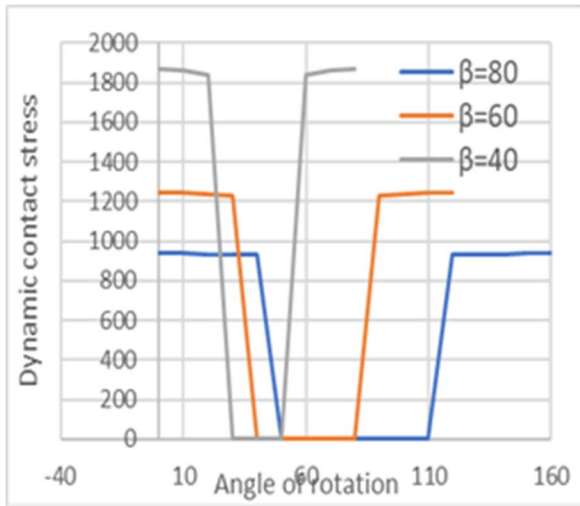
The variation of rise and return angle led to increase contact force about (289%) as shown from 1931 N to 7512 N, see **Figure 5**.

• Dynamic contact stress

Maximum contact stress value located at ( $\theta=0$ ) and has a value of 932.036 N/mm<sup>2</sup>, this value increase by reducing rise and return angle about (101%) to reach value (1866N/mm<sup>2</sup>) see, **Figure 6**.



**Figure 5:** rise and return angle effect on Dynamic force with angle of rotation



**Figure 6:** rise and return angle effect on Dynamic contact stress with angle of rotation

2. Spring stiffness

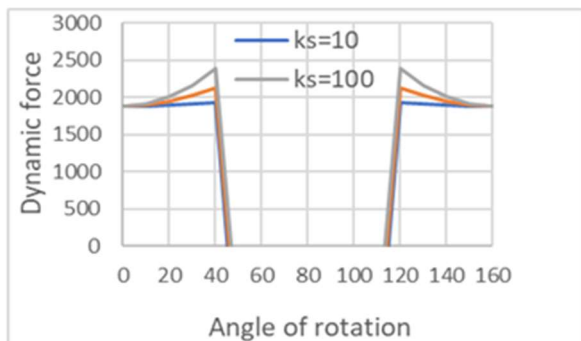
Contact force and contact stress changing by change spring force as below:

- Dynamic contact force

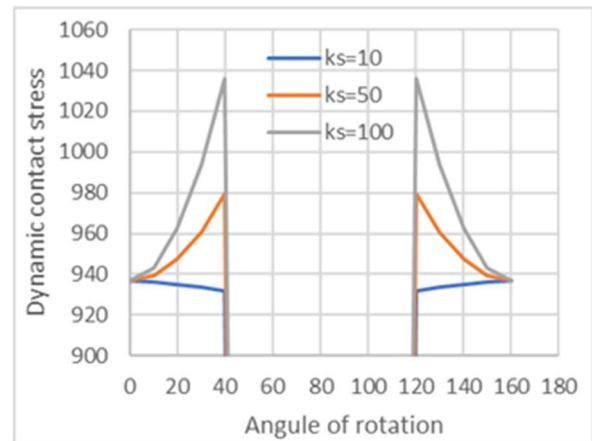
Maximum contact force indicates at  $\theta = 40$ , the variation of spring stiffness from 10 to 100 led to increase contact force about (26) % as shown **Figure 7**.

- Dynamic contact stress

Dynamic stress indicates according to equation (31), from Fig.7. notice that the dynamic stress located at  $\theta = 0$  when the value of stiffness is 10 N/mm. Dynamic stress and has value of 936.79 N/mm<sup>2</sup>, increasing of spring stiffness from 10 to 100 led to changing of contact stress location i.e, the maximum contact stress acts at  $\theta = 40$  instead of  $\theta = 0$  and increasing of contact stress value about (10)% . See **Figure 8**.



**Figure 7:** Spring stiffness variation effect on Dynamic contact force with angle of rotation.



**Figure 8:** Spring stiffness variation effect on Dynamic contact stress (N/mm<sup>2</sup>) with angle of rotation

3. Follower mass (mf)

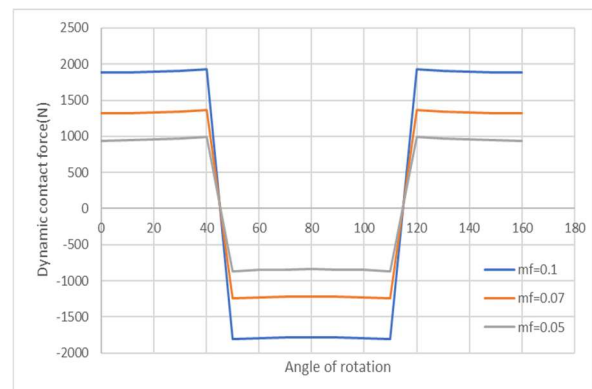
The variation of follower mass value is clear on Dynamic force and dynamic stress as following:

- Dynamic contact force

The effect of variation follower mass on dynamic force indicated in **Figure 9** it's clear that the reducing of follower mass from 0.1 kg to 0.05 kg cause to reduce contact force about (48%) .

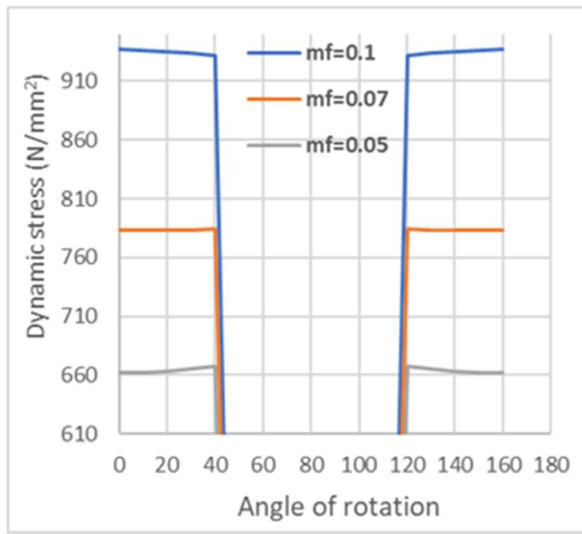
- Dynamic contact stress

Based on equation (32), **Figure 10** Indicates that the variation of follower mass value, change the position of maximum stress. It is clear that reducing the follower mass from 0.1 to 0.05 change the maximum force position from  $\theta = 0$  to  $\theta = 40$  and increase the max. Dynamic stress by about 28% from 936.79 N/mm<sup>2</sup> to 667 N/mm<sup>2</sup>.



**Figure 9:** Follower mass variation effect on Dynamic contact force with angle of rotation





**Figure10:** Follower mass variation effect on Dynamic contact stress with angle of rotation

4. Maximum lift stroke

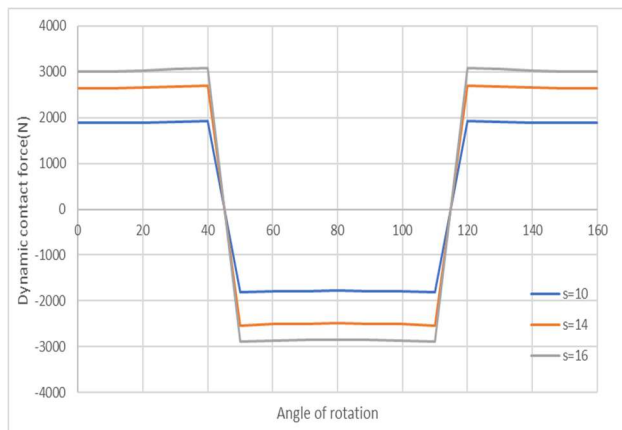
The variation of maximum stroke of cam cause to change contact force and dynamic stress that applied to cam as following:

- Dynamic force

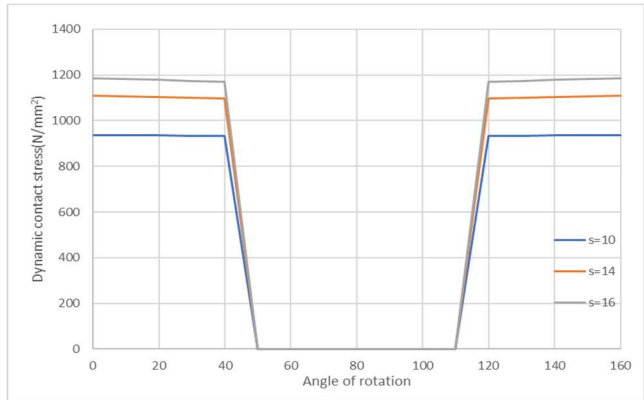
Figure 11, indicates the effect of max. lift variation on contact force, it's clear that the increasing of cam stroke from 10 mm to 16 mm led to increase contact force about (60) %.

- Dynamic contact stress

The variation of cam stroke from 10mm to 16 mm cause to increasing of maximum contact stress about (26) % and the both case maximum contact stress at  $\theta = 0$  as shown Figure 12.



**Figure 11:** lift stroke variation on contact fore with angle of rotation



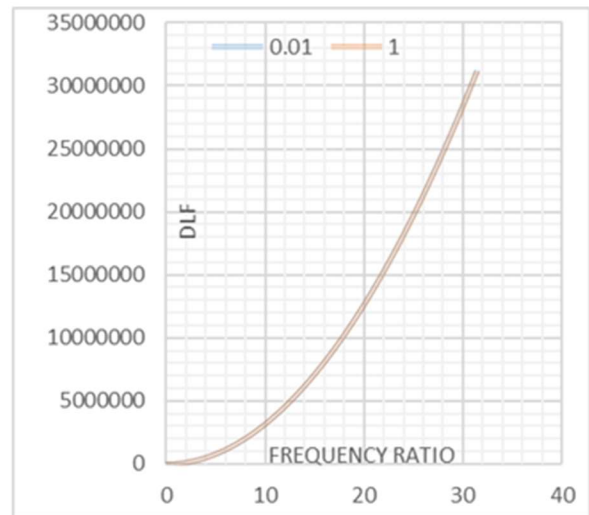
**Figure 12:** Maximum stroke variation on dynamic stress with angle of rotation

7.2 Dynamic load factor

Dynamic load factor has been calculated according to equation (29). DLF effected by damping ration and frequency ratio, it also effected by design parameter as spring stiffness and mass follower as following:

- Damping ratio and frequency ratio

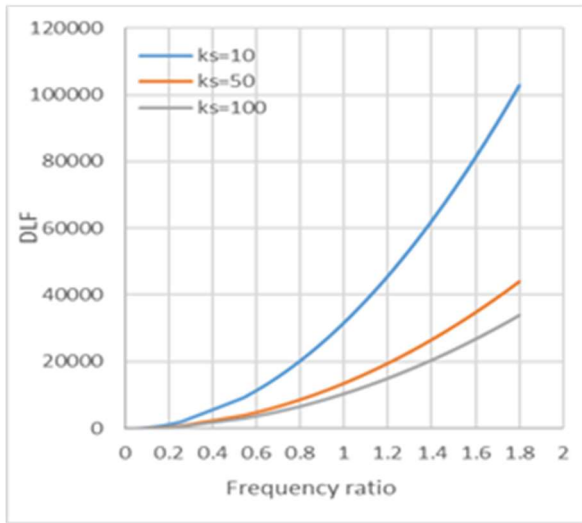
The camshaft dynamic load factor for different damping ratios has been expressed in Figure13 and it is clear that the increasing of damping ration from 0.01 to 1 lead to minimize DLF about (0.47%) in addition to that dynamic load factor increase with increasing frequency ratio.



**Figure 13:** Dynamic load factor variation with the variation of the frequencies ratio ( $\xi$ ) for different damping ratio

- Spring stiffness

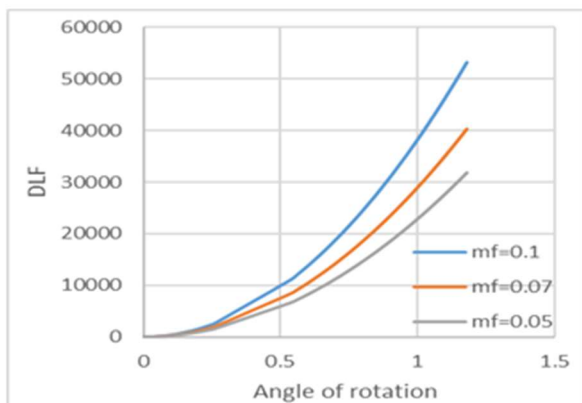
Figure 14 Shows that the variation of spring stiffness from 10 N/mm to 100 N/mm led to reduce dynamic load factor about (87) %.



**Figure 14:** Spring stiffness variation effect on Dynamic load factor with Frequency ratio

• Follower mass

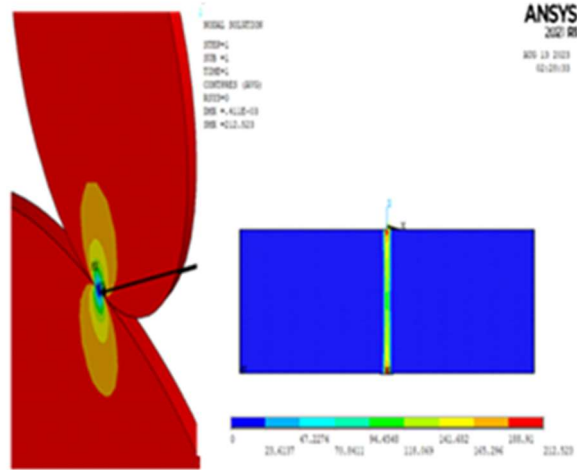
Reducing follower mass from 0.1kg to 0.05 kg led to reduce DLF about (48) % as shown **Figure 15**



**Figure15:** Follower mass variation on dynamic load factor

**7.3 Numerical result**

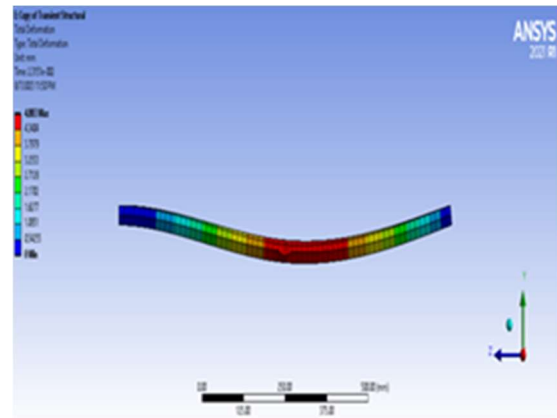
For static investigation result The FEM results in Figure 16, Analyze the induced static contact stress at the max rising position at the max. Applied load i.e. 100 N and  $\theta=80$ . The max. Contact stress is 212N/mm<sup>2</sup>. The percentage difference compared with analytical result is 1.4%.



**Figure16:** FEM solution of cam contact stress at the max. Lift stroke for F=100N. Mass variation on dynamic load factor

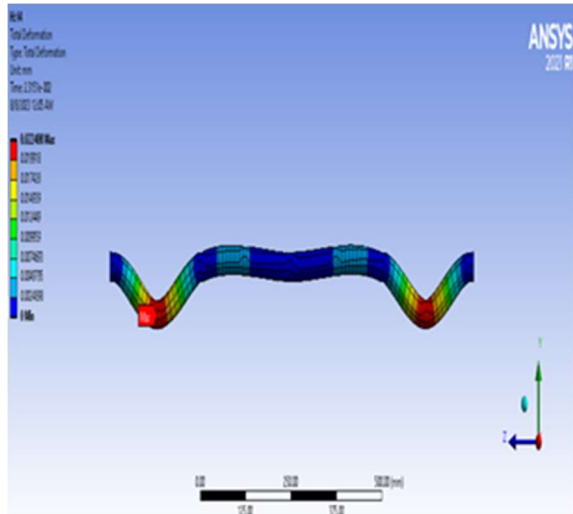
Numerical result deflection of uniform acceleration force data indicates according equation (13). Maximum dynamic deflection for camshaft has single element on mid span is (4.8) mm, see, Figure.17.

Dynamic load factor numerical solution has been calculated by dividing the numerical dynamic deformation over static deformation (0.26) mm, the numerical value of dynamic load factor is (18.3). the range of difference between the analytical and numerical results is 4.3%.

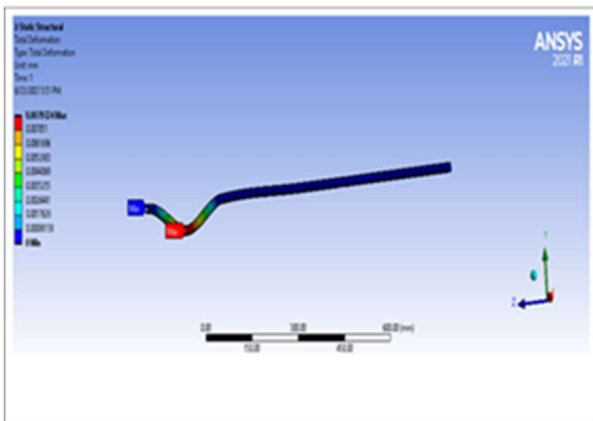


**Figure 17:** Deflection of shaft under uniform acceleration single force on the middle of shaft

**Figure 18** Figure 18, shows the deflection result of multi element shaft subjected to four uniform forces and it clear that the value of deflection is (0.022 mm), Static deformation of shaft has a value of (0.0079 mm), and see Figure 19, DLF Value is (2.7).



**Figure 18:** Deflection of shaft under four uniform acceleration forces



**Figure 19:** Static deflection of shaft under 100N static load subjected at 125mm shaft length

## 8. CONCLUSIONS

From the whole results there are some concluding remarks which can be insert such:

1. Reducing follower mass plays a major role on decreasing the contact stress.
2. Cam geometrical properties have apposite effect on dynamic contact stress reduction.
3. Dynamic load factor effected by increasing damping ratio which has positive effect on dynamic load factor minimizing.
4. For high load frequency stiffness could control the vibration.
5. Cam magnification factor couldn't be founded from the ratio of the Max. Static stress.

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### Greek symbols

$\beta$	Rising or falling angle
$\theta$	Angular position (degree)
$\nu_f$	Follower Poisson's ratio.
$\nu_c$	cam Poisson's ration.

### Nomenclature

E	Modulus of elasticity of cam and follower (N/mm <sup>2</sup> ).
I:	second moment of area (mm <sup>4</sup> )
L	length of shaft (mm).
rb	Cam base radius (mm)
rc	Cam radius of carvature
rf	Follower radius of curvature,
S	stroke of cam (mm).
b	Face width (mm)
$r'$	Velocity of follower (mm/s).
$r''$	Acceleration of follower (mm/s <sup>2</sup> ).

## صياغة معامل الحمل الديناميكي لعمود الحدبات تحت اثاره تعجيل منتظم

صفا علي محمود شكري<sup>1\*</sup>، نصير رشيد حمود<sup>2</sup>

<sup>1</sup>قسم الهندسة الميكانيكية، جامعة بغداد، بغداد، العراق، Email: safa.shukri2003m@coeng.uobaghdad.edu.iq

<sup>2</sup>قسم الهندسة الميكانيكية، جامعة بغداد، بغداد، العراق. Email: nassear\_machine@yahoo.com

\*الباحث الممثل: صفا علي محمود شكري، Email: safa.shukri2003m@coeng.uobaghdad.edu.iq

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**الخلاصة** – نظراً لأهميتها في العديد من التطبيقات الميكانيكية، كانت الحدبات موضوعاً للعديد من الأبحاث فيما يتعلق بتحسين قوة تحمل ضغوط التلامس الديناميكية والاحتكاك بناءً على تعديلات هندسة الحدبة ومتغيرات التابع. يهدف هذا العمل إلى تقييم تأثير متغيرات حدبة التعجيل المنتظمة على القوى التلامس الديناميكية ومعامل الحمل الديناميكي لعمود الحدبات. تمت صياغة معادلة الحركة بناءً على قانون نيوتن الثاني مع الأخذ في الاعتبار هندسة فص الحدبة وحركة التابع لتقييم قوة الاتصال الديناميكية أثناء سير العمل من حيث السرعة الزاوية لعمود الحدبات والزمن. يحتوي التحليل النظري على جزأين هما؛ تقييم اجهادات التلامس الساكنة والديناميكية على أساس صيغة هيرتز ومعامل الحمل الديناميكي لعمود الحدبات ذو العنصر الواحد كعتبة مدعمة من الطرفين على أساس نظرية عتبة أويلر- بيرنولي. تم اعتماد FEM لمحاكاة ضغوط التلامس الساكنة والديناميكية والانحراف الديناميكي لعمود الحدبات متعدد العناصر والتحقق من نتيجة الحل النظري. أظهرت النتائج أن زيادة نسبة التخميد وجساءة النابض لها تأثير إيجابي على تقليل عامل الحمل الديناميكي بينما زيادة كتلة التابع لها تأثير سلبي على تقليل معامل الحمل الديناميكي.

**الكلمات المفتاحية** – معامل الحمل الديناميكي , حركة بتعجيل منتظم , عمود الحدبات